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| **UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry** |

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**SPECIFICATION REFERENCES**

N7 Calculate with roots and with integer **and fractional** indices

N8 calculate exactly with fractions and **surds** …

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

A4 simplify and manipulate algebraic expressions (including those involving surds) by collecting like terms …

A5 understand and use standard mathematical formulae; …

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …

G3 … understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G4 derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; …

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras’ theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G11 solve geometrical problems on coordinate axes

G20 know the formulae for: Pythagoras’ theorem *a*2 + *b*2 = *c*2, and the trigonometric ratios sine, cosine and tan; apply them to find angles and lengths in right-angled triangles … and in two dimensional figures

G21 know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45°, 60° and 90°; know the exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60°

**PRIOR KNOWLEDGE**

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trig formulae.

Students should recall basic angle facts.

Students should understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

**KEYWORDS**

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, Pythagoras’ Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, segment, length

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| **5b. Pythagoras’ Theorem and trigonometry**  (N7, N8, N15, A4, A5, R12, G6, G20, G21) | **Teaching time**  5-7 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Understand, recall and use Pythagoras’ Theorem in 2D;
* Given three sides of a triangle, justify if it is right-angled or not;
* Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
* Find the length of a shorter side in a right-angled triangle;
* Calculate the length of a line segment *AB* given pairs of points;
* Give an answer to the use of Pythagoras’ Theorem in surd form;
* Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
* Use the trigonometric ratios to solve 2D problems;
* Find angles of elevation and depression;
* Know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45°, 60° and 90°; know the exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60°.

**POSSIBLE SUCCESS CRITERIA**

Does 2, 3, 6 give a right-angled triangle?

Justify when to use Pythagoras’ Theorem and when to use trigonometry.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Combined triangle problems that involve consecutive application of Pythagoras’ Theorem or a combination of Pythagoras’ Theorem and the trigonometric ratios.

In addition to abstract problems, students should be encouraged to apply Pythagoras’ Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach? Rounding skills will be important here when justifying their findings.

**COMMON MISCONCEPTIONS**

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer, or round their answer prematurely.

**NOTES**

Students may need reminding about surds.

Drawing the squares on the three sides will help when deriving the rule.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

To find in right-angled triangles the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras’ Theorem and trigonometry together.