

INQUIRY TEACHING

Andrew Blair proposes replacing strategy and investigational lessons with jointly-regulated inquiries, in order to harmonise method with content.

One consequence of arranging the mathematics curriculum into a hierarchical scale has been that teachers view learning as a sequential process. While the national curriculum rests on the idea of a predictable path from one level of attainment to the next, the numeracy strategy sets out the order for teachers to cover the concepts and skills that comprise those levels. Even though teachers know that learning is rarely characterised by smooth advancement, they accept responsibility for ‘closing the gap’ between students’ current and target levels by following the prescribed programme. However, teachers often begin to realise the impoverished nature of strategy lessons when asked to consider a levelled learning trajectory designed to promote student independence (Table 1).

	problem	means	solution
level 1	given	given	given
level 2	given	given	to be found
level 3	given	to be found	to be found
level 4	to be found	to be found	to be found

Table 1 (Harpaz, 2005)

In the level-1 classroom, the problem, means and solution are given by the teacher; at level 4, they are to be found by students. Rarely do numeracy lessons get beyond level 1. In order to teach a skill or underlying concept, the teacher is normally expected to pose the problem (by setting the task), give students the means (by determining the lesson structure and outlining the preferred method) and provide solutions (in order to facilitate students’ assessment of progress).

Acceptance of levelled progress leads to a distrust of teaching models that do not emphasise an ordered progression in content. It can seem inconceivable to many teachers that mathematics lessons could or, indeed, should reach level 4, in which students raise their own questions, structure their own inquiries and find solutions unknown to the teacher. For the sceptic, there seems no

guarantee that students would cover the required content knowledge after level 1. By allowing students to direct their own learning, the teacher sacrifices the possibility of a comprehensive understanding of the curriculum, and ultimately this could harm students’ chances in national examinations. Consequently, this is seen as unfair or even a dereliction of the teacher’s responsibility.

Such arguments, however, overlook the restricted and shallow nature of learning in the level-1 classrooms. Knowledge comes pre-packaged and is consumed in a way determined by the teacher. The method of learning and the content to be learned are both extraneous to the will of the student, and might, for individuals or the whole class, exist in a contradictory relationship. Hence, the method of instruction might restrict, or even block, students’ ability to learn.

In encouraging students to negotiate the structure of activities, the inquiry teacher aims to harmonise method with content. Students not only meet concepts and skills in a meaningful pursuit of answering their own or peers’ questions, they also participate in a debate about *how* to learn and *why* to learn in a certain way. While the student at level 1 remains dependent on the teacher, the student at level 4 would be developing metacognitive skills, such as monitoring and regulating thinking, that are necessary to learn autonomously.

Content

Herron (1971) proposed a four-point scale of ‘openness’ in inquiry classrooms (Table 2). His descriptions of the four levels map directly onto those of Table 1.

The inquiry classroom seldom fits neatly into these levels. At level 2, for example, students might pose a question, but need teacher guidance in structuring the inquiry. At level 3, would the inquiry be ‘open’ if the teacher provided the prompt to inquiry as opposed to the student finding the stimulus in her or his own experience?

level	description
0 confirmation or verification	Students answer a question set by the teacher, using a prescribed method and confirming a known and predictable outcome.
1 structured inquiry	The teacher poses the question and determines the structure of an inquiry; the student has the opportunity to discover relations between variables.
2 guided inquiry	The teacher provides a question, while students select or design a procedure to reach an outcome that they (and possibly the teacher) cannot foretell.
3 open inquiry	All stages of the inquiry – question, method and solution – are generated by the student from the starting point of a material stimulus or observation.

Table 2: Levels of inquiry

Ultimately, the inquiry teacher is operating on a continuum; the amount of structure or guidance offered by the teacher is contingent on students' ability to think independently and critically, which, in turn, is related to their previous participation in inquiry lessons.

The fully 'open' inquiry in the mathematics classroom seems incompatible with a national curriculum:

'No fixed and established curriculum, however well constructed, could really respond to the needs of an instructional approach that stresses students' independent learning' (Borasi, 1992: 202).

Nevertheless, a curriculum attempts, albeit in a limited and often biased way, to represent what Vygotsky (1994) called the 'final and ideal form' of development. He claimed that, through a school-based programme of instruction, students internalise 'scientific' concepts prevailing in the environment. The mathematical concept of number, for example, has been refined and perfected in a socio-historical process. It is both 'ideal' in the sense of being a model towards which student thinking should develop and 'final' in representing the end of child development.

However, Vygotsky rejected the static view of learning that his formulation might imply. Direct teaching of concepts, he contends, is 'impossible and fruitless', as students cannot internalise them in 'an unaltered, ready-made form' (Vygotsky, 1986: 150 and 159). Indeed, he speculated that scientific concepts are the medium through which self-awareness and self-regulation develop: 'Reflective consciousness comes to the child through the portals of scientific concepts' (Vygotsky, 1986: 171). Thus, in interacting with the environment as the *source* of development, students orient towards socially meaningful concepts and, in doing so, develop their capacity to think metacognitively.

In an inquiry classroom, concepts are introduced in order to illuminate a mathematical

process that all participants have the chance to direct. Yet it is the responsibility of the teacher as 'a more experienced knower' to select students' ideas that provide a link to mathematical concepts (Goos, 2004: 263). In order to promote the learning of such concepts, the inquiry teacher can introduce a 'prompt' that delineates the terrain of the inquiry but is sufficiently open to allow discussion on its structure and to permit different trajectories. It is no contradiction to incorporate a specific part of the curriculum, as an expression of a socially-developed concept, into the planning of a stimulus to inquiry. It is improbable, though, that a class would focus exclusively on a single content objective. By leaving the questions or conjectures arising from the prompt to students' curiosity, the teacher adopts a flexible approach in guiding the inquiry towards legitimate mathematical knowledge.

Method

A second reaction to inquiries, along with the concern about coverage, is to dismiss them as just another name for investigations. The key distinction between the two, however, lies in the method of instruction. Inquiries are flexible and responsive to the context; investigations follow the rigid structure of experimentation in science classrooms. As a typical case of investigations, GCSE students were given the 'borders' shape (Figure 1) as a starting point for coursework and expected to use the following procedure:

- draw more shapes in a sequence
- isolate variables
- tabulate results
- identify patterns
- test predictions
- make generalisations
- link the structure of the shapes to the generalisation.

This inductive process of 'discovering' a general statement from empirical data is well-established in

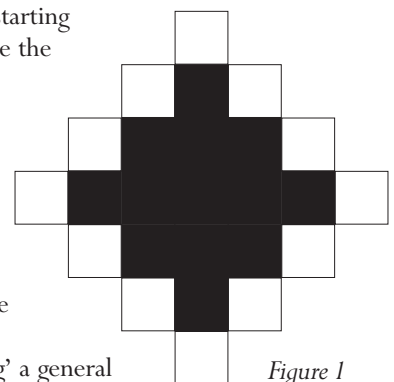


Figure 1

the investigational classroom. On Herron's scale (Table 2), investigations would be characterised as *structured* inquiries (level 1) because both problem and method are prescribed. In the case of the mark schemes for GCSE coursework, the outcome was determined as well. So, whatever the intentions of teachers, mathematical investigations have been standardised and, consequently, fallen into disrepute as an indicator of students' ability to reason and communicate independently.

The process of turning investigations into a set routine, moreover, is not simply the result of an unfortunate misinterpretation on the part of teachers or examination boards, but exposes a flaw in the pedagogy of discovery underlying the investigative approach. The method easily accommodates *empirical* induction, through which the student can reach a general statement from a number of observations and thereby predict a next case. However, the method lacks a mechanism for promoting *mathematical* induction, through which the student can prove the truth of the generalisation for *all* cases. Often the process of proof requires a prior knowledge of concepts that are combined, often intuitively, in a specific way. In the discovery classroom, there is no mechanism to introduce concepts and theoretical propositions to prove general assertions, especially if, for the student, those concepts remain 'undiscovered'.

In a typical investigation, students are guided towards 'discovering' a mathematical relationship. By drawing squares on the sides of right-angled triangles, for example, students might be expected to 'discover' Pythagoras' Theorem from the areas of those squares. At the start of the investigation, the teacher might ascertain if the class understands the concept of area and can calculate the area of a square. However, teachers are often placed in a quandary if a student fails to discover the theorem. The dilemma of whether or not to tell is compounded when teachers take a constructivist position on learning. If individuals construct their own meanings, there is no guarantee that a student will develop a correct understanding of the theorem simply by being told. In reality, teachers have no option but to guide a student, through a series of hints, to the realisation of the correct relationship.

The weakness of the discovery lesson is overcome in the inquiry classroom by the mechanism of *joint regulation*, which occurs through negotiation. The teacher encourages students not only to decide upon and request knowledge necessary to reach a better understanding of a stimulus, but also to suggest a structure for the inquiry

linked to their identified strengths and weaknesses. Joint regulation provides a mechanism for the teacher, firstly, to steer the public evaluation of proposed activities towards mathematical methods and, secondly, to introduce 'ideal' concepts linked to those activities.

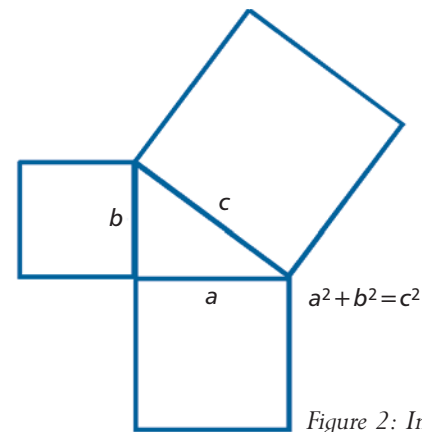


Figure 2: Inquiry prompt

Curiously, the 'answer' from a discovery lesson often represents an effective prompt for an inquiry, such as a diagram stating Pythagoras' Theorem (Figure 2). Students have the opportunity to ask questions or make statements about the diagram, often exposing conceptual weaknesses as they do so. The class might aim to clarify its meaning through dialogue and then decide on a period of exploration, deducing the theorem's applicability to right-angled triangles only. As any decision to draw more examples similar to the diagram has been made through joint regulation (rather than being a necessary stage of an empirically-rooted investigation), students have felt empowered to put forward further suggestions. Negotiations based on the teacher's proposal to change elements of the diagram, for example, have led students to replace the squares with other shapes, such as similar rectangles. From empirical observations, they have gone on to induce a general relationship using the new shapes. However, the skill of the inquiry teacher lies in the ability to link comments made by students to structures of inquiry that include or lead to mathematical induction. In the case of the prompt in *figure 1*, the teacher might highlight the limited nature of empirical reasoning and, if judged appropriate, guide a class towards a proof of the theorem. Thus, this inquiry carries the potential for deduction and induction of both kinds.

Method with content

Although rewarding and exciting, inquiry lessons place a burden on teachers. Not only must they be prepared to redistribute authority to students,

References

- Blair, A. (2007) The *all, most and some* of learning objectives, *MT202*, 20-22
- Borasi, R. (1992) *Learning Mathematics through Inquiry*, Heinemann
- Goos, M. (2004) Learning mathematics in a classroom community of inquiry, *Journal for Research in Mathematics Education*, 35(4), 258-91
- Harpaz, Y. (2005) Teaching and learning in a community of thinking, *Journal of Curriculum and Supervision*, 20(2), 136-57
- Herron, M. D. (1971) The nature of scientific enquiry, *The School Review* 79(2), 171-212
- Leikin, R. and Rota, S. (2006) Learning through teaching: A case study on the development of a mathematics teacher's proficiency in managing an inquiry-based classroom, *Mathematics Education Research Journal*, 18(3), 44-68
- Vygotsky, L. S. (1986) *Thought and Language*, The MIT Press
- Vygotsky, L. S. (1994) The problem of the environment; In Van der Veer, R. & Valsiner, J. (eds) *The Vygotsky Reader*, Blackwell

teachers should also become proficient in conducting inquiries before they can expect improvements in students' active learning (Leikin and Rota, 2006). Key to that proficiency is the flexibility to harmonise method with content. That inquiries, even those that begin with the same prompt, have the potential to be very different reflects the high number of factors faced by the teacher. Those factors include:

- the mathematical terrain suggested by the prompt;
- students' initial questions and conjectures;
- the negotiated aims of the lesson (see Blair, 2007);
- the degree to which students identify and understand concepts related to the prompt;
- students' previous experience of structuring inquiries and their awareness of possible learning trajectories;
- students' ability to regulate the learning environment.

The methods proposed in inquiry lessons go through a process of evaluation and justification before being selected. A variety of structures have resulted from attempts to harmonise method with content by individual classes. The prompt

$$24 \times 21 = 42 \times 12$$

has led to student expositions on multiplying two double-digit numbers, individual exploration in search of similar equations, and group or whole-class attempts to find a complete set through employing concepts of ratio or algebraic manipulation. One class faced with the prompt

$$x + y = 4$$

generated horizontal groups conducting separate inquiries into, for example, straight-line graphs or changing the subject of a formula, ending with presentations to the rest of the class. When inquiring into the prompt

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z},$$

another class, however, opted for vertical groupings studying at different levels of abstraction. Students decided whether they needed to learn or revise how to add fractions (by choosing an activity under the guidance of the teacher), or whether they could search for integer solutions or even devise an equation that links the three variables.

Inquiries also demand structures linked to different forms of mathematical thinking. To reach a comprehensive understanding of the prompt

'The sum of two fractions equals their product'

requires the insight that the condition is fulfilled by improper fractions. This realisation has been arrived at by individual students in the supportive climate of a class discussion. A prompt on the

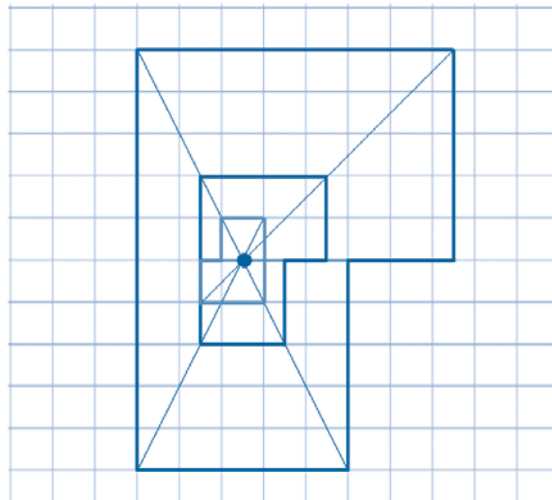


Figure 3

concept of enlargement (Figure 3), however, has involved students in taking different perspectives on the original and enlarged shapes. These perspectives have emerged in small groups of students working on the same diagram.

A student's conceptual learning is facilitated by a harmonious method of learning. In teacher-directed strategy lessons, there is little guarantee that a method permits students to reach an 'ideal' understanding. In the investigative classroom, the learning experience is rigidly structured, leaving students the unenviable responsibility to discover a conceptual relationship. It is only the inquiry classroom that offers students a mechanism to harmonise conceptual learning with the method of learning. The mechanism is joint regulation of the structure of the inquiry. Table 3 summarises the contentions of this article.

	Herron's level	concept	method	harmonisation
strategy lesson	0	determined by teacher	determined by teacher (through set task)	unpredictable
investigation	1	to be discovered by student	determined by teacher (empirical induction)	possible; no mechanism to overcome disharmony
inquiry	2 or 3	suggested by prompt	agreed in negotiation	likely; mechanism of joint regulation to ensure harmony

Table 3

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