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# DEVELOPING AUTOMATICITY IN MULTIPLICATION FACTS: INTEGRATING STRATEGY INSTRUCTION WITH TIMED PRACTICE DRILLS

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***Abstract.*** Automaticity in math facts has been of considerable interest to special educators for decades. A review of the intervention literature suggests at least two common approaches to developing automaticity in facts. One is grounded in the use of strategies for teaching facts, the other emphasizes the use of timed practice drills. Recent research indicates that students might benefit from an integration of these two approaches. This experimental study contrasted an integrated approach (i.e., strategies and timed practice drills) with timed practice drills only for teaching multiplication facts. Participants were 58 fourth-grade students with a range of academic abilities. Fifteen of the students in the study had IEPs in math. Results indicated that both approaches were effective in helping students achieve automaticity in multiplication facts. However, students in the integrated approach generally performed better on posttest and maintenance test measures that assessed the application of facts to extended facts and approximation tasks. These results have implications for teaching a range of skills and concepts that are considered important to overall mathematical competence in the elementary grades.

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Information-processing theory supports the view that automaticity in math facts is fundamental to success in many areas of higher mathematics. Without the ability to retrieve facts directly or automatically, students are likely to experience a high cognitive load as they perform a range of complex tasks. The added processing demands resulting from inefficient methods such as counting (vs. direct retrieval) often lead to declarative and procedural errors (Cumming & Elkins, 1999; Goldman & Pellegrino, 1987; Hasselbring, Goin, & Bransford, 1988). Potential difficulties extend well

beyond operations on whole numbers. Finding common multiples when adding fractions with unlike denominators or factoring algebraic equations are but two examples from secondary-school mathematics where automaticity in math facts can facilitate successful performance.

Advocates of contemporary approaches to mathematics, ones that tend to place more emphasis on conceptual understanding and problem solving than on computational skills, see an important place for automaticity in math facts. Isaacs and Carroll (1999), for

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example, note that automaticity is essential to estimation and mental computations. These skills, particularly the ability to perform mental computations (e.g., make approximations based on rounded numbers such as 10s and 100s), are central to the ongoing development of number sense. In an effort to reach consensus on the current state of K-12 mathematic education, Ball and colleagues (2005) also affirm the importance of automaticity in math facts.

Unfortunately, decades of research show that academically low-achieving students as well as those with learning disabilities (LD) exhibit considerable difficulty in developing automaticity in their facts. Difficulties and delays are apparent from the beginning of elementary school. These students fail to retrieve facts directly when presented in isolation or when embedded in tasks such as multidigit computations. Research on primary-grade students indicates that students with LD are more likely to rely on counting strategies than direct retrieval when working single-digit fact problems (Geary & Brown, 1991; Hanich, Jordan, Kaplan, & Dick, 2001; Hoard, Geary, & Hamson, 1999). These students also tend to make more retrieval and counting errors on simple addition problems than their non-LD peers.

Results of Goldman and her colleagues' research on second- through sixth-grade students with LD show that these students tend to rely heavily on counting over direct retrieval methods (Goldman, Pellegrino, & Mertz, 1988). When compared with non-LD peers, students with LD often employ suboptimal strategies when retrieving facts. For example, if given a problem like  $2 + 9$ , students with LD generally do not employ the more sophisticated strategy of commuting and then adding the numbers in order to derive 11. Instead, they tend to count up from 2 to 11, a method that often results in an incorrect sum. Goldman et al. (1988) conclude that elementary students with LD are delayed in their ability to learn facts automatically, and suggest that this delay can be addressed through systematic practice. This finding is consistent with the results of other research (e.g., Geary, 1993), which suggest that interventions are necessary for students with LD in order to ensure that they can retrieve facts automatically by the end of elementary school.

### ***Teaching Facts Through Strategies***

Brownell and Chazal's (1935) early work in math fact instruction initiated a debate over the best approach to teaching facts that has continued to the present day. Their work questions the traditional emphasis in schools on rote memorization, which, if done excessively, can reinforce students' use of immature methods for answering fact problems such as the counting-up strategy described above. Isaacs and

Carroll (1999) echo this concern, emphasizing that students naturally develop strategies for learning math facts if given the opportunity. Research supporting the natural development of strategies may be found for addition and subtraction (Baroody & Ginsburg 1986; Carpenter & Moser 1984; Resnick 1983; Siegler & Jenkins 1989) as well as more recent work in the area of multiplication (Anghileri, 1989; Baroody, 1997; Clark & Kamii, 1996; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005).

As a consequence of this research, a number of educators emphasize the use of explicit strategy instruction over traditional rote learning when teaching math facts. Methods vary from the use of visual displays such as ten frames and number lines (Thompson & Van de Walle 1984; Van de Walle, 2003) to more general techniques such as classroom discussions where students share fact strategies with their peers (Steinberg, 1985; Thornton, 1990; Thornton & Smith, 1988).

Specific recommendations for multiplication strategies vary considerably in the way fact strategies are linked to a broader ability to perform mental calculations. For example, some special education researchers (e.g., Miller, Strawser, & Mercer, 1996) stress basic rules for multiplication as they relate to math facts (e.g., multiplication by 0 or 1, the commutative property). These rules are frequently taught in math classes. Others (e.g., Chambers, 1996; Garnett, 1992; Thornton, 1990) recommend a wider array of strategies and focus on patterns that are easier to learn. They suggest that doubles, times five, times nine patterns, and square numbers are easier for students to learn than facts such as  $4 \times 8$  and  $6 \times 7$ . Van de Walle (2003) also makes this observation in his discussion of strategy instruction for facts.

The link between facts and mental calculations is more evident in recent discussions of number sense (author; Kilpatrick, Swafford, & Findell, 2001; Sowder, 1992). French (2005) shows how derived fact (e.g.,  $6 \times 7 = 6 \times 6 + 6$ ) and counting backwards for 9s strategies (e.g.,  $8 \times 9 = 8 \times 10 - 8$ ,  $9 \times 9 = 9 \times 10 - 9$ ) can be applied to calculations involving 2 x 1 digit numbers. For example, students with good number sense calculate  $99 \times 9$  by converting the problem to  $100 \times 9 - 9$ . Strategies such as "split - add" for  $8 \times 4$  involve splitting the problem into two smaller problems (i.e.,  $8 \times 2 + 8 \times 2$ ) and then adding the products. A similar decomposition logic can be applied in the form of distributive multiplication (e.g.,  $27 \times 2 = 25 \times 2 + 2 \times 2$ ). Other strategies such as doubling and halving (e.g.,  $32 \times 5 = 32 \times 10 \div 2$ ) also expand a student's ability to compute mentally exact answers to multidigit multiplication problems. The extent to which French's prescriptions apply to academically low-achieving students is not clear.

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Math educators argue that emphasis on strategies helps students organize facts into a coherent knowledge network (Isaacs & Carroll, 1999; Rathmell, 1978), thus facilitating long-term retention and direct recall. Strategic instruction facts can even include instruction on extended facts (e.g.,  $3 \times 4$  extends to  $30 \times 4$ ). The link of facts to extended facts ostensibly helps students in estimation and mental computation tasks. Finally, Sherin and Fuson (2005) argue that strategic knowledge remains in many students even though they are able to answer multiplication fact problems within 3 seconds per fact parameters. Students may use a combination of strategies (i.e., “hybrids”) when answering relatively difficult facts such as  $6 \times 7$ . Their research shows that for a fact such as this, students may rapidly employ a derived fact strategy.

### ***Timed Practice Drills***

Cumming and Elkins (1999) point out that many educators and researchers make the unwarranted assumption that strategies – either developed naturally or through explicit instruction – invariably lead to automaticity. However, research cited above (e.g., Geary, 1993; Goldman et al., 1988) indicates that students with LD do not develop sophisticated fact strategies naturally. Furthermore, empirical research on strategy instruction in math facts for students with LD is limited, and the results are mixed in terms of the effective development of automaticity (see Putnam, deBettencourt, & Leinhardt, 1990; Tournaki, 2003).

Timed practice drills as a method for developing automaticity offer a clear alternative to strategy instruction for academically low-achieving students and students with LD (Ashcraft & Christy, 1995; Geary, 1996). Special education research supports time drills that often include pretesting and systematic review (Burns, 2005; Koscinski & Gast, 1993; Morton & Flynt, 1997; Stein, Silbert, & Carnine, 1997).

Hasselbring, and his colleagues (Hasselbring et al., 1988; Hasselbring, Lott, & Zydney, 2005) present one of the most sophisticated formats for timed practice drills. In addition to pretesting, they present facts through families. For example, a fact like  $4 \times 7 = 28$  is linked instructionally to  $28 \div 4 = 7$  because of its “fact family” relationship (i.e., 4, 7, 28 are numbers used in a multiplication and division fact family). Their computer-based approach introduces new facts targeted for instruction in small sets, thus reducing the amount of new information to be learned. Further, the program systematically reviews these facts over time as a method of maintaining automaticity. The use of distributed practice in the program is often cited as a key principle for teaching topics such as math facts

(Anderson, Reder, & Simon, 1996; Kameenui, Carnine, Dixon, Simmons, & Coyne, 2001).

### ***Integrating Strategy Instruction and Timed Practice Drills***

Cumming and Elkins’ research (1999) suggests that a middle-ground position for teaching facts to academically low-achieving students and students with LD consists of integrating strategy instruction with frequent timed practice drills. Results of their recent research indicate that instruction in strategies alone does not necessarily lead to automaticity. Frequent timed practice is essential. However, strategies help increase a student’s flexible use of numbers, and for that reason, Cumming and Elkins advocate the use of strategy instruction for all students through the end of elementary school.

Recent international comparative research in mathematics indicates that fact strategies are a consistent feature of instruction for elementary-grade Asian students (Fuson & Kwon, 1992; Ma, 1999). As argued above, strategy instruction also can benefit the development of estimation and mental calculations. In this respect, strategy instruction helps develop number sense, a topic of emerging interest in the special education literature (author; Baroody & Coslick, 1998; Gersten & Chard, 1999).

### ***Purpose of the Study***

This study was designed to examine the impact of an integrated approach to teaching multiplication facts to fourth-grade students with and without LD. The intervention draws on the instructional design features described by Hasselbring and his colleagues (Hasselbring et al., 1988; Hasselbring et al., 2005) for timed practice drills as well as contemporary approaches to strategy instruction in math facts (Garnett, 1992; Isaacs & Carroll, 1999; Ma, 1999; Sherin & Fuson, 2005; Thornton, 1990; Van de Walle, 2003). This means that strategy instruction also includes an emphasis on the link between facts and extended facts.

The following four research questions guided the study:

1. Would an integrated approach to teaching facts lead to greater automaticity in facts than timed practice only?
2. Would an integrated approach lead to superior performance on extended facts and approximation tasks?
3. Would extended practice on multidigit computational problems in the comparison condition lead to significant differences for the timed practice only group?

The study attends to some key concerns raised in recent discussions of scientifically based research. In addition to the random assignment of students to inter-

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vention and comparison conditions, two important limitations commonly found in quantitative research (see Gersten, Lloyd, & Baker, 2000) are addressed.

First, many experimental and quasi-experimental studies use weak or ill-described interventions as comparison conditions. In this study, we employed a well-documented method for teaching facts, the direct instruction approach for timed practice drills, as the comparison condition. Direct instruction has considerable support as a broad-based instructional intervention in special education (Kroesbergen & Van Luit, 2003; Swanson, Hoskyn, & Lee, 1999).

Second, empirical studies can be biased because they employ measures that are only sensitive to what is being taught to the intervention group. In this study, we employed measures that are sensitive to instruction in both the intervention and comparison conditions.

Finally, the study examined the impact of the intervention in a mainstreamed, instructional environment. This is important because many students who may need additional mathematics assistance do not receive them for a variety of reasons (e.g., they do not qualify for special education services, the primary remedial and special educational services are committed to reading instruction, students may qualify for special education in math but receive the majority of their instruction in mainstreamed or inclusive settings).

## METHODS

### *Participants and Setting*

The study was conducted in a suburban school district in the Pacific Northwest. Two classrooms with a total of 58 fourth-grade students in the same elementary school participated. Fifty-seven percent of the students in the school were on free or reduced-cost lunch. According to district data, 36% of the students were minorities – primarily black and African American – and 20% were receiving special education for mathematics.

Mean grade equivalence on the Math Computations subtest of the Iowa Test of Basic Skills (ITBS; Hoover, Nieronymus, Dunbar, & Frisbie, 1996) for the 58 participants was 3.6 (the 41st percentile). This indicated that students, on average, were approximately one year behind grade level when the study was conducted. Further demographic information on the participants is presented in Table 1.

The 15 special education students who participated in the study had been identified through district and statewide criteria for learning disabilities. All of the students were of normal intelligence, and a discrepancy model including intelligence was used to determine placement in special education. The mean grade equivalence on the ITBS computation for these students was 2.9, or the 13th percentile.

Scores on the Math Computations subtest of the ITBS and a 40-item multiplication fact test, the Common Multiplication Facts Test described later, were used as the basis for matching and then randomly assigning students to conditions. Results from the Common Multiplication Facts Test indicated to researchers that all students could be placed at the same level of facts at the beginning of the study. Thirty students were assigned to the intervention group, which also included eight students who had IEPs for LD in math. Twenty-eight students were assigned to the comparison group; of this group, seven students had IEPs for LD in math. Performance on the ITBS Math Computations subtest for both groups was comparable (Integrated Strategies mean = 23.93, Time Practice Only mean = 25.54). A *t*-test was performed on this measure indicated non-significant differences between groups ( $t_{(56)} = .73; p = .48$ ).

## MATERIALS

**Integrated group.** Students in the intervention group were taught fact strategies that have been described in the literature on multiplication facts cited earlier. Many of these strategies have been incorporated into *Transitional Mathematics, Level 1* (Woodward & Stroh, 2004a) and *Fact Fluency and More!* (Woodward & Stroh, 2004b) curricular materials, which were used as the basis for daily instruction.

Facts were divided into relatively easy facts and those that have generally been considered more difficult. A range of empirical and descriptive literature was used to support this distinction. Hence, rule-based facts like 0s and 1s, doubles, perfect square, and times 5s, 9s and 10s have tended to be easier for students to learn at a level of automaticity (Goldman et al., 1988; Sherin & Fuson, 2005; Van de Walle, 2003; Wood & Frank, 2000; Wood, Frank, & Wacker, 1998).

The literature has recommended that more difficult facts be taught through derived fact or doubling and doubling-again strategies. For example, multiplication facts such  $6 \times 7$  and  $7 \times 8$  in the materials used for the study were taught through a derived strategy ( $6 \times 7 = 6 \times 6 + 6$ ,  $7 \times 8 = 7 \times 7 + 7$ ). An example of the visual representation of the derived fact strategy for  $6 \times 7$  may be found in Figure 1.

Facts like  $6 \times 4$  were taught through a doubling and doubling-again strategy (e.g.,  $6 \times 4$  is shown as  $6 \times 2 = 12$  and then  $12 \times 2 = 24$ ). The materials used with the Integrated group included number lines and arrays of blocks projected on the overhead as a way of helping students visualize the fact strategies.

A controlled practice scheme similar to one employed by Hasselbring et al. (2005) was used to organize the daily timed practice worksheets. New facts were pre-

sented in small sets (i.e., an average of 7 facts and no more than 10 facts for each strategy). The timed practice worksheets were organized around a specific strategy. Each daily timed practice worksheet contained 40 facts. Fifty percent of the facts were review facts; the remaining 50 percent were a random presentation of

the new facts. The review facts functioned as distributed practice.

Other materials were used to teach the connection between the single-digit facts described above and extended facts (e.g., 4 x 2 extends to 40 x 2 and 400 x 2). The instructional materials included worksheets

**Table 1**  
*Participant Characteristics*

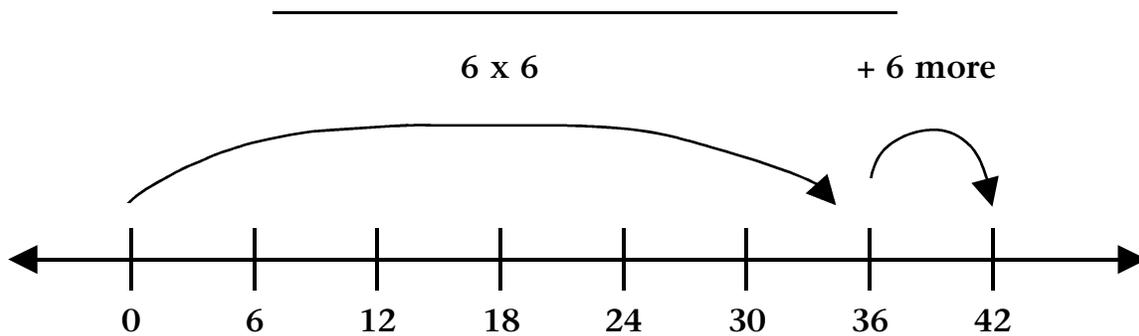
Variable	Group	
	Integrated Strategies	Timed Practice Only
N <sup>a</sup>	30	28
Non-LD	22	21
LD <sup>b</sup>	8	7
Age	113(5.03) <sup>c</sup>	113(5.39)
Non-LD	112.86(5.56)	113.01(5.95)
LD	113.38(3.42)	113.86(3.39)
ITBS Computations	23.93(6.50)	25.54(5.60)
Non-LD	26.82 (4.48)	28.05(3.64)
LD	16.00(4.07)	18.00(2.94)
Common Facts Pretest	27.03(5.42)	27.18(7.14)
Non-LD	27.86(5.58)	28.43(7.17)
LD	24.75(4.53)	23.43(6.02)
Hard Multiplication Facts Pretest	11.97(9.35)	16.32(11.15)
Non-LD	13.00(10.48)	19.24(11.25)
LD	9.13(4.52)	7.57(4.32)
Attitude Survey	47.93(10.41)	49.79(6.60)
Non-LD	48.73(11.30)	49.81(7.17)
LD	45.75(7.69)	49.71(4.92)
Gender Ratio	16F /14M	13F /15M
Non-LD	14F /8M	10F /11M
LD	2F /6M	3F /4M
Racial Balance	15 Black/13 White/2 Asian	12 Black/16 White
Non-LD	10 Black/10 White/2 Asian	8Black/13 White
LD	5 Black/3 White	4 Black/3 White

<sup>a</sup> 57% of students were on free or reduced-cost lunch.

<sup>b</sup> All students were of normal intelligence; a discrepancy model was used to determine placement in special education.

<sup>c</sup> All scores in parentheses represent standard deviations.

**Figure 1.** A derived fact strategy for  $6 \times 7$ .



where students completed 20 randomly ordered single-digit facts and extended facts. This was an untimed activity, and it was intended to help students generalize their knowledge of facts. Number line and array block overhead transparencies like the one shown in Figure 2 were used when the teacher and students discussed extended facts.

Students also learned the partial product algorithm (see Figure 3) for  $2 \times 1$  and  $3 \times 1$  digit problems. This algorithm was used to reinforce extended facts. Students worked sets of 10 problems using this algorithm.

Visual representations such as the one shown in Figure 4 were projected on an overhead, and were designed to help students with approximation skills. The teacher used these number lines to show how two- and three-digit numbers can be rounded to their nearest 10s or 100s. Students were given worksheets where they rounded  $2 \times 1$  and  $3 \times 1$  computational problems and computed approximate answers using extended facts.

Finally, the teacher presented a number of word problems such as the one shown in Figure 5. These problems were presented to the class as a whole, and different students took turns demonstrating how they could use extended facts to find an approximate answer to the problem.

**Timed practice only group.** Methods for students in the comparison group were based almost entirely on the controlled practice approach found in the literature, particularly the use of direct instruction for teaching math facts and the traditional multiplication algorithm (see Stein et al., 1997). This approach contained many of the instructional principles described in the Hasselbring et al. (2005) research. New facts were taught sequentially

(i.e., start with the 1s, then the 2s), and previously taught facts were reviewed on a daily basis and incorporated into the daily timed practice drills.

The direct instruction approach used for multidigit multiplication worksheets stressed a clear presentation of the traditional algorithm. Problems at the beginning of the study were  $2 \times 1$  digit problems and continued gradually up through  $2 \times 2$  and  $3 \times 1$  digit problems. The materials were designed to provide systematic review of previous problem types as students moved hierarchically from easier to harder problems.

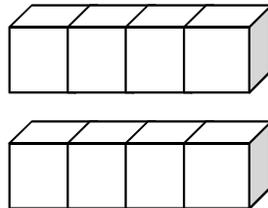
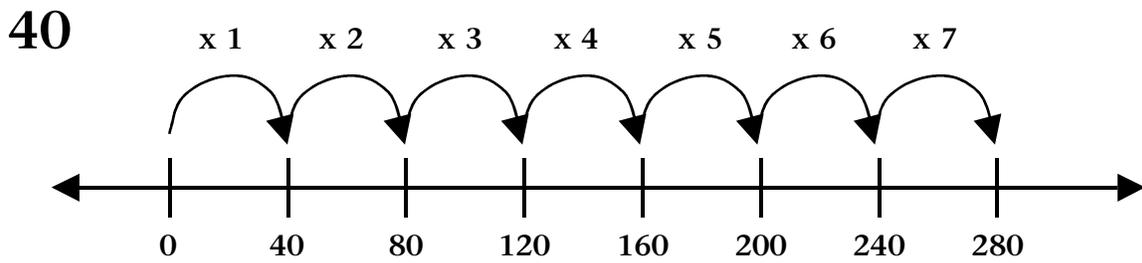
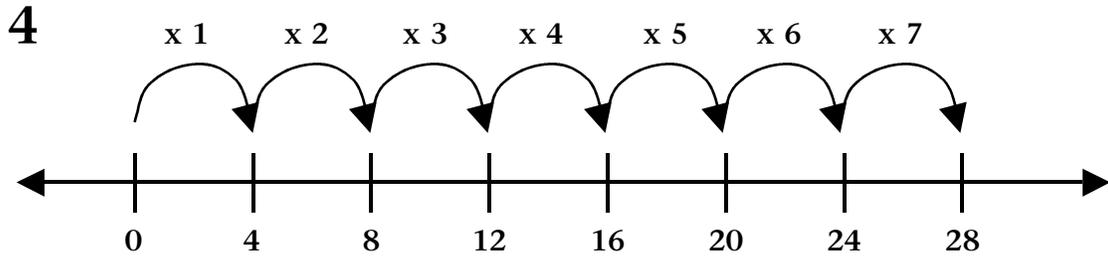
### **Procedures**

Students were matched and randomly assigned based on results of pretest measures (i.e., Math Computations subtest of the ITBS and the Common Multiplication Facts Test). Scores were rank ordered, and pairs of students at the top of each list were assigned randomly to each condition. At the beginning of each instructional session, the students went to their respective classrooms (i.e., intervention or comparison).

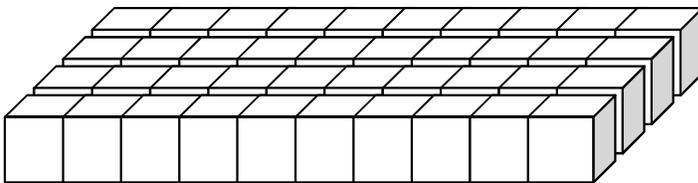
Students in both groups were taught for 25 minutes per day, five days a week, for four consecutive weeks. Researchers made every effort to control the amount of instructional time for each group so that it was the same each day. The teachers who participated in the research had extensive experience with both the instructional methods and the materials employed in each condition. They met with the research team prior to the study and reviewed materials as well as key teaching components for their respective method (i.e., the integrated and the timed practice only).

To ensure fidelity of implementation a researcher on the project observed classroom instruction on a

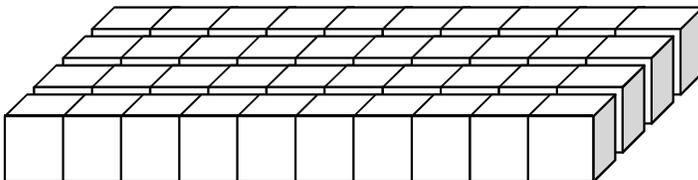
Figure 2. Links between facts and extended facts using block arrays and number lines.



$$4 \times 2 = 8$$



$$40 \times 20 = 800$$



**Figure 3.** The partial product algorithm and extended facts.

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$$\begin{array}{r} 341 \\ \times 2 \\ \hline 2 \\ 80 \\ + 600 \\ \hline 682 \end{array}$$

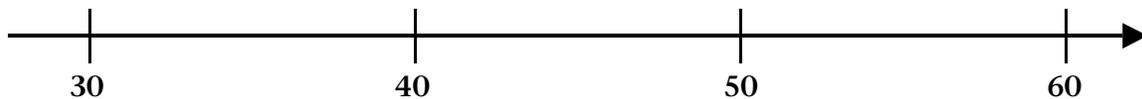
biweekly basis using used a checklist consisting of key instructional techniques for each intervention. The checklist and observational notes were used as the basis for discussions with the two teachers. This biweekly discussion helped ensure that each method was implemented consistently and correctly. Finally, the teachers in the study were counterbalanced midway through the study to control for teacher effects.

**Integrated group.** Daily instruction for the integrated group followed three distinct phases. The first phase was either an introduction of new fact strategies or a review of the previously taught strategies. The teacher devoted extra time to this phase of the lesson on the days when students were shown a new strategy (e.g., the derived fact strategy for  $6 \times 7$ ). The teacher used arrays or number lines on the overhead to visualize the strategy

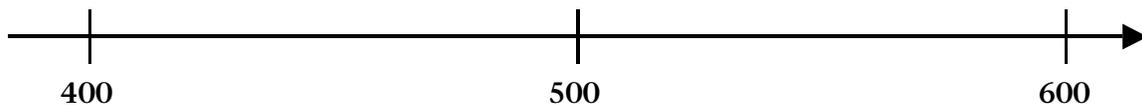
**Figure 4.** Number lines for teaching locating and rounding numbers.

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**“Find 43 and round to the nearest 10.”**



**“Find 478 and round to the nearest 100.”**



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(review Figure 1). Students were encouraged to discuss the strategy and contrast it with previously taught strategies. For example,  $9 \times 5$  could be considered a “times 5” strategy, but it was also part of the “times 9 strategy.”

On other days, the teacher used the overhead to discuss facts that represented previously taught strategies. Arrays or number lines were used to visualize the strategy. Students were not required to memorize the strategies and use them in a rote manner on any of the worksheets used during the intervention.

The second phase of daily instruction was a 2-minute timed practice drill. Materials for this phase have been described above. Following the 2 minutes, the teacher dictated answers to the facts, and students corrected their own work. They circled any incorrect facts and wrote correct answers after the teacher finished dictating the answers.

The teacher collected the daily worksheets and monitored the progress of the group as they approached automaticity in a particular strategy. When 70% of students achieved 90% correct on a strategy, the teacher shifted instruction to the next strategy. Automaticity was considered to be 36 correct problems within the 2-minute time period. No additional instruction was provided for students who were not at the 90% accuracy level.

The final phase of daily instruction varied. Periodically, students were taught the relationship between facts and extended facts. The teacher used arrays and number lines as visual models to foster this connection (review Figure 2). On other days, the teacher presented extended facts in the context of partial product multiplication (review Figure 3). Individual students

explained how to compute a multiplication problem using the algorithm.

Approximation skills were taught by first emphasizing rounding to 10s and 100s. Number lines such as the one shown in Figure 4 were the basis for helping students locate numbers and visualize the rounding process. The teacher used worksheets where students rewrote  $2 \times 1$  and  $3 \times 1$  problems and then approximated the answers. Finally, word problems like the kind shown in Figure 5 were the subject of classroom discussion and problem solving. Each of these activities helped reinforce the role of facts and extended facts in number-sense tasks such as multiplication computations, mental calculations, and approximations.

**Timed practice only group.** Daily instruction for this group also followed three distinct phases. The first phase involved either the introduction of new facts or a systematic review of previously taught facts. The teacher used a prepared set of facts for each lesson. When students were taught new facts, the teacher introduced the fact and its answer, and then the students were asked the answer to the fact (e.g., “ $9 \times 3$  is 27. What is  $9 \times 3$ ?”). In the case of previously taught facts, the group was simply asked the answers to fact questions (e.g., “What is  $7 \times 4$ ?”). In both instances, the group gave choral responses. These formats are considered key features of direct instruction (see Stein et al., 1997).

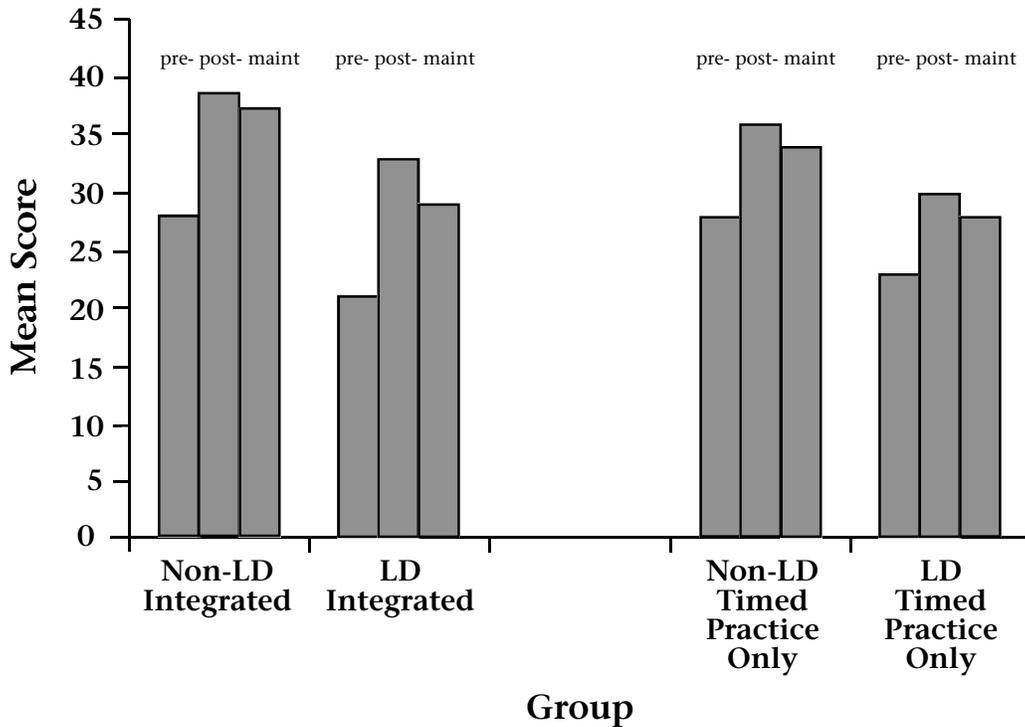
The second phase of instruction involved a 2-minute timed practice drill. Materials for this component of the instruction were described above. At the end of the drill, the teacher dictated the correct answers to the facts and students corrected their own work circling any incorrect facts and writing in correct answers.

**Figure 5.** Sample word problems presented to the integrated group.

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“Let’s say that you were traveling across country on an airplane. The plane left from San Francisco to New York. The flight takes 5 hours. Early in the flight the pilot makes this announcement, “we are traveling at 513 miles per hour, and we expect to land in New York at 6:35 pm. About how far is it from San Francisco to New York?”

**Figure 6.** Pre-, post-, and maintenance scores for non-LD and students with LD for the common multiplication facts.



When shifting to instruction on new facts, the teacher followed the same procedures as the teacher in the integrated condition. When 70% of students achieved 90% correct on a strategy, the teacher focused on the next set of facts. Also, no additional instruction was provided for students who were not at the 90% proficiency level.

The third phase of instruction involved worksheet practice on computational problems using the traditional multiplication algorithm. The teacher modeled how to compute new types of problems as they moved from 3 x 1 to 2 x 2 digit problems over the course of the study. The teacher also reminded students to use their knowledge of facts as they completed problem sets. Computational problems were restricted to those involving facts that had been taught up to this point in the study. Hence, students did not work computational problems that involved any facts that they had not learned as part of the study. Once students had completed each set of problems, the teacher dictated

answers to the problems and students corrected any errors.

#### **Measures**

A range of measures were developed to assess students' understanding of multiplication facts and related mathematical skills. The ITBS was the standardized measure of computational skills used. For all other computational measures, an independent group of 22 fourth-grade students was used to calculate test-retest reliability for each measure. All of the measures had test-retest reliability above .90. General attitudes toward mathematics were also assessed.

**Math computations subtest of the ITBS.** The Math Computations subtest of the ITBS (Hoover et al., 1996) is a standardized measure of basic computational skills. The primary purpose of this subtest is to assess competence in numeration as well as the ability to perform arithmetic operations on whole numbers. The measure was used to match students prior to the study, and it also functioned as a covariate in the data analyses.

**Facts measures.** Knowledge of multiplication facts was assessed in three ways. First, a 40-item test of the Common Multiplication Facts was developed. The “times 0” and “times 1” facts were excluded from the test. This test was useful because it included many facts that students knew at the beginning of the study and, thus, enabled researchers to see how well previously taught facts were maintained at a level of automaticity. The measure also enabled researchers to place students in both conditions at the same instructional level at the beginning of the study.

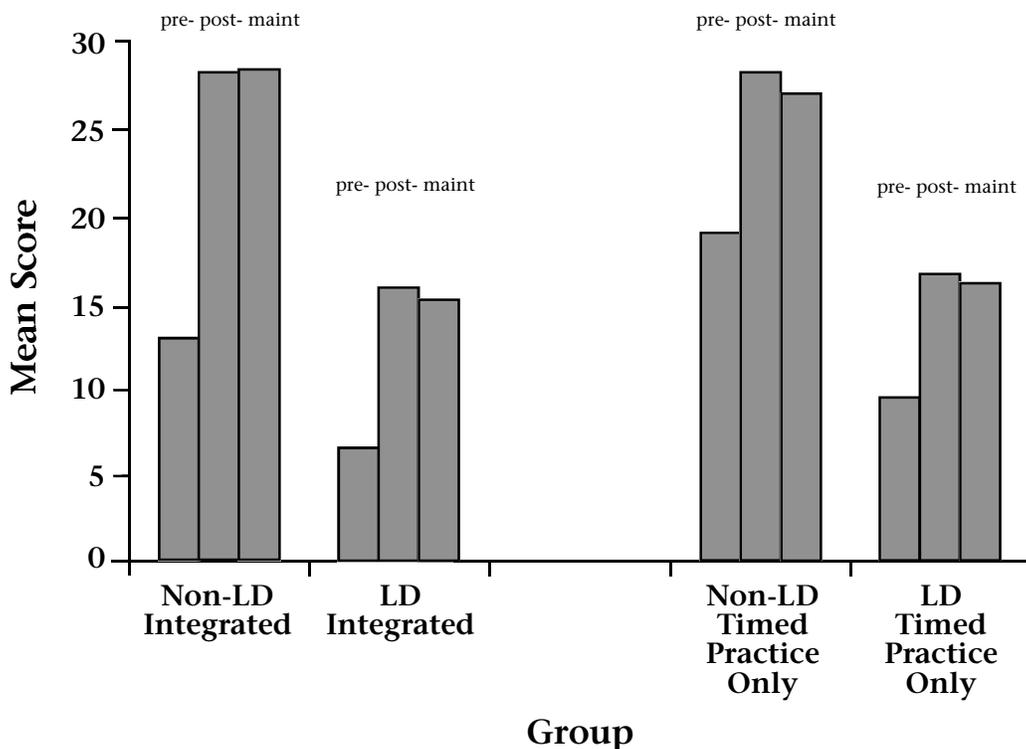
A second measure was entitled the Hard Multiplication Facts. These were 40 facts that students typically had difficulty answering correctly in the pretest phase. In fact, a mean overall performance of 36% for all of the students on the pretest helped corroborate the difficulty level of these facts. Hence, these facts became a primary focus of each intervention.

Both measures – the Common Multiplication Facts

and the Hard Multiplication Facts – were administered as pretests, posttests, and 10 days following the posttest as maintenance tests. There was no school in the intervening 10 days due to a spring break. It was deemed unlikely that any of the students received instruction in math facts during that period of time. Students were given 2 minutes to complete each of the 40 item measures.

A third measure was called the Extended Facts Test. The 15 items making up this test assessed fluency in computing extended facts. The difficulty of the items were restricted to 2 x 1 digit problems (e.g., 30 x 4, 60 x 6), and students were given 90 seconds to complete the test. The time limit was determined based on pilot work prior to the study. This measure was timed because it is unlikely that there would have been any differences between groups if students had been given unlimited time to complete the 15 items. This measure was administered as a posttest only.

**Figure 7.** Pre-, post-, and maintenance scores for non-LD and students with LD for the hard multiplication facts.



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**Multiplication computation measures.** Two other posttest measures were used to assess knowledge of multiplication. The first was the Computations Test. The 15 items on this measure were a mix of 3 x 1 and 2 x 2 digit problems. There was no time limit for completing this measure.

The final measure was the Approximations Test. This 15-item test asked students to round 2 x 1 and 3 x 1 digit computational problems to produce an approximate answer. Students were given 90 seconds to complete the test. A time limit was used to determine how quickly students could round two- and three-digit numbers and then apply extended facts.

**Attitude measure.** In order to determine their attitudes toward math, students were given a 16-item general survey entitled Attitudes Toward Math. This survey assessed general attitudes toward the subject and the extent to which students thought they were good at problem solving, worked well with numbers, and so forth. Each question involved a 1-4 Likert scale, yielding a maximum score of 64 points. This pretest and posttest measure was group administered. The measure has been used in previous research (author); test-retest reliability was .91.

The attitude survey provided a global indication of student attitude toward math. The test asked students to rate how good they were in math, if they thought of themselves as good problem solvers, and other questions such as their ability in multiplication and math facts. It was also another way for researchers to determine if students perceived one approach more favorably than the other. This was an exploratory measure, and the researchers did not anticipate significant differences between groups in attitudes toward math to emerge over the period of the intervention.

## RESULTS

Analyses of the data are grouped according to facts, computational abilities in multiplication, and attitudes toward mathematics. The Math Computations subtest score of the ITBS was used as the covariate in all analyses of covariance (ANCOVA). Cohen's *d* was used as a measure of effect size for all between-group comparisons that were statistically significant. Furthermore, the data have been disaggregated into students with and without LD in order to show the relative impact of the intervention for each group. The sample size of students with LD in each group did not permit the use of inferential statistics; therefore, the effects of the different instructional methods on these students and their non-LD peers are presented descriptively. These comparative data shed light on the effectiveness of the respective instructional methods for all students in each class.

## Fact Measures

**Common facts.** A 3 x 2 ANCOVA (time by group) was performed on the Common Multiplication Facts Test. Descriptive statistics for students by group and time (i.e., each of the repeated measures) may be found in Table 2. Data were analyzed using the Wilks' lambda criterion. Results of this analysis suggested a significant difference between condition groups ( $F_{(1,55)} = 6.38, p = .01, d = .68$ ). There was a significant interaction for time and group ( $F_{(2,54)} = 3.89, p = .02$ ).

A series of *t*-tests were performed on the pretest, posttest, and maintenance tests. There were nonsignificant differences between groups on the pretest ( $t_{(56)} = -.09$ ), but significant differences between groups on the posttest ( $t_{(56)} = 2.22, p = .02, d = .27$ ) and again on the maintenance test ( $t_{(56)} = 2.20, p = .02, d = .27$ ).

Mean percent correct on posttest and maintenance tests for both groups was at a relatively high level, particularly in comparison to the pretest scores. The data suggest that the Integrated group remained at a mastery level from posttest to maintenance test (i.e., at or above 90%). This was not the case for the Timed Practice Only group.

A comparison of students with and without LD by group on the Common Multiplication Facts Test is presented in Table 2 as well as graphically in Figure 6. As illustrated, the performance of both groups of students rose substantially on the common facts measure by the time of the posttest. It should be noted that students with LD in both groups were still below the mastery level (i.e., 90% correct). Performance for all subgroups declined from the post- to the maintenance test, with the least proportionate decline for non-LD students in the Integrated group.

**Hard facts.** A 3 x 2 ANCOVA (time by group) was performed on the Hard Multiplication Facts Test. Descriptive statistics for students by group and time are presented in Table 3. Data on these repeated measures were also analyzed using the Wilks' lambda criterion. Results showed nonsignificant differences between groups ( $F_{(1,55)} = .13, p = .72$ ) and near-significant differences within groups ( $F_{(1,55)} = 3.83, p = .06$ ). There was a significant interaction for time and group ( $F_{(2,54)} = 4.61, p = .01$ ).

The source of the significant interaction occurred between the pretest and posttest. A 2 x 2 ANCOVA was performed on this phase of the repeated measures; it indicated significant differences within groups ( $F_{(1,55)} = 4.52, p < .05$ ). Changes in mean scores help explain the significant interaction. The mean pretest score for Integrated students was 11.97 (30% correct) compared to 16.32 (41% correct) for the Timed Practice Only group. Posttest mean scores for both groups rose to virtually identical levels at 25.76 and 25.46 (64% correct),

**Table 2**  
**Means, Standard Deviations, and Percent Correct for the Common Multiplication Facts**

Group	N	Pretest			Posttest			Maintenance Test		
		M	SD	Mean % Correct	M	SD	Mean % Correct	M	SD	Mean % Correct
Integrated	30	27.03	5.42	68	37.57	3.95	94	36.13	5.37	90
Timed Practice Only	28	27.18	7.14	68	34.18	7.27	86	32.36	7.80	81
<b>Students Without LD</b>										
Integrated	22	27.86	5.58	70	38.91	1.79	97	37.55	3.55	94
Timed Practice Only	21	28.24	7.16	71	35.57	6.88	90	33.76	7.64	84
<b>Students with LD</b>										
Integrated	8	21.29	4.46	53	32.86	5.61	82	29.43	6.63	73
Timed Practice Only	7	22.88	5.87	57	30.13	6.90	75	27.50	6.90	69

**Table 3**  
**Means, Standard Deviations, and Percent Correct for the Hard Multiplication Facts**

Group	N	Pretest			Posttest			Maintenance Test		
		M	SD	Mean % Correct	M	SD	Mean % Correct	M	SD	Mean % Correct
Integrated	30	11.97	9.35	30	25.76	9.80	64	25.23	9.55	63
Timed Practice Only	28	16.32	11.15	41	25.46	10.86	64	24.61	10.90	62
<b>Students Without LD</b>										
Integrated	22	13.00	10.47	33	28.41	8.68	71	27.91	8.19	70
Timed Practice Only	21	19.24	11.25	48	28.23	11.02	71	27.14	11.31	70
<b>Students with LD</b>										
Integrated	8	6.57	3.91	16	15.71	6.92	39	14.88	6.82	37
Timed Practice Only	7	9.25	4.13	23	16.63	4.53	42	16.12	5.11	40

respectively. The differences between groups for this phase of the study was near a significant level ( $F_{(1,55)} = 3.55, p = .07$ ).

The comparison of students without LD and those with LD by group on the Hard Facts Test shown in Table 3, as well as graphically in Figure 7, indicated a considerable gap in acquisition rate of these facts over time. While students in each group showed measurable gains from pre- to posttest, students with LD in both groups were well below their peers with respect to fact mastery. No subgroup of students achieved mastery of these facts by the end of the four-week intervention. Interesting, decline from post- to maintenance on these facts was comparable – and marginal – across subgroups (i.e., on average, approximately 2% correct).

**Extended facts.** An ANCOVA performed on the Extended Facts Test indicated significant differences favoring the Integrated group ( $F_{(1,55)} = 16.38, p < .001, d = 1.08$ ). Means, standard deviations, and percent correct for this posttest may be found in Table 4. Percent correct indicated that, on average, the Integrated students were at a mastery level of performance. Again,

this was not the case for the students in the Timed Practice Only group.

Table 4 also shows mean performance differences between subgroups on the extended facts, with only the students without LD in the Integrated group scoring above the mastery level criterion. Consistent, measurable differences were also found between non-LD and students with LD on this test within each group as well as noticeable differences between students with LD in the Integrated and Timed Practice Only conditions.

#### **Multiplication Measures**

**Computations measure.** An ANCOVA was performed on the Computations Test. Results for this posttest measure indicated nonsignificant differences between groups ( $F_{(1,55)} = 2.84, p = .10$ ). However, percent correct as shown in Table 5 suggested considerably higher levels of performance on the part of the Timed Practice Only students.

All subgroups scored below what would commonly be considered mastery-level performance on this measure (i.e., 80 – 85% correct). The range of mean

**Table 4**

***Means, Standard Deviations, and Percent Correct for the Extended Facts Test***

<b>Group</b>	<b>N</b>	<b>M</b>	<b>SD</b>	<b>Mean % Correct</b>
Integrated	30	13.47	2.11	90
Timed Practice Only	28	10.78	3.90	72
<b>Students Without LD</b>				
Integrated	22	13.95	1.76	93
Timed Practice Only	21	11.76	3.88	78
<b>Students with LD</b>				
Integrated	8	11.14	2.79	74
Timed Practice Only	7	8.88	3.09	59

**Table 5**  
**Means, Standard Deviations, and Percent Correct for the Computations Test**

Group	N	M	SD	Mean % Correct
Integrated	30	8.43	5.38	56
Timed Practice Only	28	10.67	3.05	71
<b>Students Without LD</b>				
Integrated	22	9.45	5.20	63
Timed Practice Only	21	11.42	2.63	76
<b>Students with LD</b>				
Integrated	8	5.00	5.00	33
Timed Practice Only	7	9.00	2.39	60

scores also reflected the respective interventions, with non-LD and students with LD in the Timed Practice Only group scoring near or above the non-LD students in the Integrated group. Mean performance also indicated that students with LD in the Integrated group performed substantially below the three other subgroups.

**Approximations measure.** A final ANCOVA was performed on the Approximations Test. The results indicated significant differences favoring the Integrated group ( $F_{(1,55)} = 39.37, p < .001, d = 1.68$ ). Descriptive statistics for this posttest measure are shown in Table 6. Differences between groups in mean percent correct were noticeable, with 83% correct for the Integrated group compared to 51% correct for the Timed Practice Only group.

Mean performance on this measure also reflected the relative impact of the instructional methods used in each condition. That is, the greater emphasis placed on approximation skills in the Integrated group was apparent in the fact that the mean scores of the non-LD and

students with LD in this group were higher than those of students in the Timed Practice Only group. Also, Timed Practice Only students with LD scored considerably below the other three subgroups in terms of mean performance.

#### **Attitudes Toward Mathematics Survey**

A 2 x 2 ANCOVA (time by group) was performed on the two repeated measures administered to Integrated and Timed Practice Only students. Descriptive statistics for students by group and ability for time (i.e., pretest to posttest) are presented in Table 7. Data were analyzed using the Wilks' lambda criterion. Results of this analysis suggested a nonsignificant difference between groups ( $F_{(1,55)} = .43$ ). There was also a nonsignificant interaction for time and group ( $F_{(1,55)} = .02$ ). However, a significant difference was found within groups ( $F_{(1,55)} = 6.42, p = .01, d = .68$ ). Descriptive statistics shown in Table 7 indicate a modest rise in the direction of an improved attitude toward mathematics, one which was comparable for both groups.

The change in attitude toward mathematics was roughly comparable over time for all subgroups of students. Students in all groups began the study with a relatively positive attitude toward math (i.e., approximately 75%) and grew only slightly. Students with LD in the Integrated group showed the greatest improvement in attitude.

### DISCUSSION

This study contrasted two approaches to teaching multiplication facts to academically low-achieving students and students with LD in mathematics at the fourth-grade level. Both methods were effective in raising the mean performance level on a mix of math facts (i.e., common facts) to mastery or near mastery levels, and these levels dropped only slightly during the maintenance phase of the study. Cohen's *d* analyses, which indicated small effect sizes, favored the Integrated group overall on the development of automaticity in these math facts.

Both groups improved considerably in their knowledge of the harder multiplication facts. However, the

restricted four-week time frame for the intervention period meant that students in neither group achieved mastery levels of performance. Posttest and maintenance test scores for both groups were virtually identical, with mean success levels of approximately 63%.

The differences in performance levels between students with and without LD on the facts measures – particularly the hard facts – revealed the challenge of moving all students in a classroom forward at the same rate. Even though all students improved from pre- to posttest, students with LD in both groups lagged considerably behind their peers. The design of the study contributed to this result because of the decision to control the amount of time devoted to fact instruction for all students.

The most discernible differences between groups appeared with respect to the instructional activities that followed the facts instruction each day. Integrated students had the opportunity to see and discuss connections between basic facts, extended facts, the partial product algorithm, and methods for approximating

**Table 6**  
*Means, Standard Deviations, and Percent Correct for the Approximations Test*

<b>Group</b>	<b>N</b>	<b>M</b>	<b>SD</b>	<b>Mean % Correct</b>
Integrated	30	12.40	2.58	83
Timed Practice Only	28	7.57	4.46	51
<b>Students Without LD</b>				
Integrated	22	13.00	2.31	87
Timed Practice Only	21	8.76	4.43	58
<b>Students with LD</b>				
Integrated	8	10.00	3.06	67
Timed Practice Only	7	5.50	4.21	37

**Table 7**  
**Means and Standard Deviations for the Attitude Towards Math Survey**

Group	N	Pretest		Posttest	
		M	SD	M	SD
Integrated	30	47.93	10.41	51.90	10.31
Timed Practice Only	28	49.79	6.60	53.54	5.59
<b>Students Without LD</b>					
Integrated	22	48.73	11.30	51.81	10.00
Timed Practice Only	21	49.81	7.17	53.62	6.27
<b>Students with LD</b>					
Integrated	8	45.42	9.02	52.43	8.44
Timed Practice Only	7	48.00	5.21	52.13	3.48

answers to multiplication problems. As a consequence, their performance on the Extended Facts and Approximation Tests was significantly higher than that of students in the Timed Practice Only group. This was reflected in large effect sizes for both measures favoring the Integrated group.

In contrast, students in the Timed Practice Only group were given many more opportunities to practice 3 x 1 and 2 x 2 multiplication problems using the traditional algorithm. This added practice resulted in higher levels of performance, even though the differences between groups on the Computations posttest were statistically nonsignificant.

Finally, students in both groups tended to have positive feelings about math and the intervention. Mean scores on the Attitudes Toward Math Survey rose slightly over the course of the four-week intervention, with means for both groups near 52 out of a maximum possible 64 points, or 81%. The within-group differences for the two groups were at the level of medium effect sizes based on Cohen's *d*.

### **Limitations of the Study**

Group-administered timed measures such as the ones used in this study can be an efficient way of capturing data for subsequent quantitative analysis. However, as some researchers have argued (e.g., Baroody, 1997; Sherin & Fuson, 2005), the boundary between automaticity and the continued use of multiplication strategies may be less distinct than the automaticity literature generally suggests. Specially, several students who were able to answer math facts within the 3-second per fact criterion may have continued to use a mix of direct retrieval and strategic methods for answering facts. In their taxonomic analysis of multiplication and its development in elementary-aged students, Sherin and Fuson (2005) referred to this phenomenon as "hybrid" strategies. Qualitative interviews and detailed observations could have revealed the extent to which even the most competent students in this study continued to use hybrid strategies, particularly on certain types of problems (e.g., 6 x 7, 8 x 4). As Siegler (1996) noted, even the most competent elementary students

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vary in their response to mathematical tasks, depending upon the specific characteristics of problems that they are asked to solve.

A second limitation of the study involves the limited time frame. Cumming and Elkins (1999) suggest that longitudinal research is needed to determine how well academically low-achieving students maintain automaticity and the extent to which strategic instruction might be beneficial.

Finally, the decision to move on to new facts based on the performance of 70% of the students in class introduced a problem that has traditionally plagued instruction on topics like math facts in heterogeneous environments. That is, the lowest achieving students did not receive the added practice at other times in the day that would have helped them keep pace with the rest of the class. Future research should explore the effects of controlled, added practice for academically low-achieving students and those with LD.

### **Implications for Practice**

Recent national policy documents endorse automaticity in facts as important to overall mathematical competence in the K-12 education. However, this skill is now embedded in a more ambitious context than it has been in the past (Ball et al., 2005; Kilpatrick et al., 2001). While automaticity in facts is still relevant to proficiency in traditional algorithms, automaticity has become important to estimation, mental calculation, and approximation skills. These latter skills are part of the development of what has often been referred to as “number sense” (see Greeno, 1991; Kilpatrick et al., 2001; Sowder, 1992).

Results from this study indicate that an integrated approach and timed practice drills are comparable in their effectiveness at helping students move toward automaticity in basic facts. If educators were only considering facts as a foundation for traditional algorithm proficiency, either method would probably suffice. Yet, the educationally significant differences between groups found on the extended facts and approximations tests should encourage special educators to consider how strategy instruction can benefit students’ development of number sense.

These findings are consistent with Cumming and Elkins’ (1999) recommendations that strategy instruction be implemented through elementary school with all students. They are equally consistent with the thinking of math educators who place significant emphasis on problem solving and conceptual understanding (e.g., Isaacs & Carroll, 1999). Furthermore, recent analyses of mental computation skills indicate that low-achieving students have a greater tendency to rely on narrow methods – ones that reflect traditional

paper-and-pencil algorithms – than their more capable peers (Foxman & Beishuizen, 2002). An expanded focus on mental computations and approximations could address this difference. The recent emphasis on number sense skills does not diminish the importance of proficiency in traditional computational algorithms. However, the extent to which facts are taught solely for this purpose should be reconsidered.

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#### **NOTE**

This research was sponsored by a grant from the U.S. Department of Education, Office of Special Education Programs, Grant #H327A030053.