Numerical Cognition: Age-Related Differences in the Speed of Executing Biologically Primary and Biologically Secondary Processes

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Groups of younger and older adults were administered numerical and arithmetical tasks that varied in the extent to which they assess evolved versus culturally specific forms of cognition, termed biologically primary *and* biologically secondary abilities, *respectively. Componential analyses of solution times suggested that younger adults are faster than older adults in the execution of biologically primary processes. For biologically secondary competencies, a pattern of no age-related differences or an advantage for older adults in speed of processing was found. The results are consistent with the view that there has been a cross-generational decline in arithmetical competencies in the United States and are discussed in terms of models of age-related change in cognitive performance.*

A number of recent studies of the arithmetical abilities of younger and older American adults suggest no age differences or an elderly advantage for speed of executing two elementary processes, arithmetic fact retrieval and borrowing in complex subtraction (Allen, Ashcraft, & Weber, 1992; Allen, Smith, Jerge, & Vires-Collins, 1997; Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991; Sliwinski, Buschke, Kuslansky, Senior, & Scarisbrick, 1994). The results of these studies stand in sharp contrast to (a) the more typical finding of a general age-related decline in the speed of

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information processing (Cerella, 1990; Salthouse, 1996) and (b) a pattern of younger Chinese adults outperforming older Chinese adults on timed paper-and-pencil arithmetic tests (Geary, et al., 1997; Geary, Salthouse, Chen, & Fan, 1996). Our goals in the current study were to reconcile this conflicting pattern of results and, at the same time, to introduce a framework for studying age-related change in evolved and culturally specific forms of cognition, termed *biologically primary* and *biologically secondary abilities*, respectively. The first section below provides an overview of relevant cognitive arithmetic research (see Ashcraft, 1995), whereas the second presents an overview of primary and secondary abilities and a description of how this demarcation might be used in cognitive aging research. The two final sections describe primary and secondary abilities in the numerical and arithmetical domains and outline the empirical features of this study.

COGNITIVE ARITHMETIC

The first subsection presents cognitive arithmetic research as related to issues in cognitive aging, and the second presents the proposed resolution to the conflicting patterns noted above and the major hypothesis of this study.

Cognitive Arithmetic and Cognitive Aging

One of the more consistent findings in the cognitive aging literature is that older adults process information more slowly than younger adults (Cerella, 1990; Myerson, Hale, Wagstaff, Poon, & Smith, 1990; Salthouse, 1991, 1992, 1996; Salthouse & Babcock, 1991). This pattern is termed *general slowing* and is typically attributed to age-related changes in the neurobiological systems that support cognitive processes. These changes are often conceptualized as global and, as such, should affect the speed of information processing in all domains, including arithmetic (Myerson et al., 1990), although recent research suggests that the degree of slowing varies for lexical (i.e., word processing) and nonlexical (e.g., memory scanning, spatial processing) tasks (Hale & Myerson, 1996; Hale, Myerson, Faust, & Fristoe, 1995; Lima, Hale, & Myerson, 1991). The speed advantage of younger adults appears to be much larger for nonlexical than lexical processing.

In the area of arithmetic, the study of age-related differences in processing speed is complicated by a number of factors. First, there are often differences in the strategies people use to solve one arithmetic problem or another, even in adulthood (Geary & Wiley, 1991; LeFevre, Sadesky, &

Bisanz, 1996). The same person might solve one simple subtraction problem by counting and solve another problem by retrieving the answer from semantic memory (Geary et al., 1993). Averaging data over trials on which different strategies have been used for problem solving can create biased estimates of reaction time (RT) patterns (e.g., Siegler, 1989) and, given this, any age difference in the pattern of strategic use can make the interpretation of any age-related difference in RTs difficult.

Second, the solving of arithmetic problems involves several component processes—such as retrieving facts from memory or borrowing (as in 54 —8)—that might differ in theoretically important ways. Arithmetic fact retrieval is very likely to involve lexical processes, in that arithmetic facts appear to be represented in the same memory system that supports word retrieval (Dehaene & Cohen, 1991; Geary, 1993). Borrowing, in contrast, is a procedural skill and would therefore appear to be classifiable as a non-lexical process. The pattern of age-related differences in fact retrieval and borrowing might then be applicable to the lexical–nonlexical distinction noted above. Either way, assessments of RTs averaged over component processes—as is often done in cognitive aging research—might result in a biased estimate of aging-related differences in processing speed. The assessment of age-related differences in arithmetical processing is further complicated by schooling and cohort effects (Geary, Bow-Thomas, Liu, & Siegler, 1996). Geary, Bow-Thomas, et al. demonstrated that schooling greatly affects the speed of arithmetic fact retrieval, and Schaie (1996) has demonstrated important cohort effects for arithmetic in the United States.

In the United States, simple and complex arithmetic skills appear to have been at their peak for individuals who received their primary education before the second world war, dropped for those who received their primary education just after the war, and dropped again for individuals who received their primary education after the mid-1960s (Schaie, 1996). Consistent with this pattern is the finding of comparable performance of younger and older American adults, and sometimes an advantage for older adults, on tests of computational arithmetic (Geary, Salthouse, et al. 1996). In contrast, younger adults in mainland China consistently outperform their older peers on the same tests (Geary et al., 1997; Geary, Salthouse, et al., 1996). The overall pattern suggests that older American adults have had considerably more experience with computational arithmetic than younger American adults and that if they were equated on experience, then younger adults would outperform older adults on arithmetic tests in the United States, as in China.

One issue related specifically to age-related differences in speed of borrowing requires resolution. Geary et al. (1993) found a positive age effect for borrow speeds, whereas Salthouse and Coon (1994) found a negative age effect. These conflicting results might be explained by Geary et al.'s use of a production task, where participants are presented with a problem to solve and must generate an answer (e.g., 56 —9 —?), and Salthouse and Coon's use of a verification task, where participants determine whether the presented answer is correct or incorrect (e.g., 56 —9 = 47). The use of different procedures is important, because it appears that the speed and accuracy of verifying presented answers is influenced by the participants' overall familiarity with the problem (i.e., it is a recognition task), whereas the production of an answer is more heavily dependent on memory retrieval and computational processes (Campbell & Tarling, 1996; Zbrodoff & Logan, 1990). Alternatively, Salthouse and Coon suggested that the Geary et al. results might have been due to the use of a highly educated and therefore select sample of older adults.

Hypothesis

One goal of this study was empirically to replicate the Geary et al. (1993) finding for borrow speeds and the more general finding of no age-related differences in speed of arithmetic fact retrieval (Allen et al., 1992; Geary et al., 1993; Geary & Wiley, 1991; Sliwinski et al., 1994). Conceptually, the goal was to do so in a way that tested our hypothesis that these cognitive arithmetic findings are due to a schooling-related cross-generational decline in arithmetical competencies in the United States (Geary et al., 1997).

One method that can be used to assess this possibility is to compare groups of younger and older American adults on arithmetical competencies that are strongly affected by schooling, that is, secondary abilities, and related numerical competencies that do not appear to be as strongly affected by schooling, that is, primary abilities (Geary, 1995). If our hypothesis is correct, then a pattern of no age-related differences or an elderly advantage for school-taught arithmetical abilities should be found, along with a concurrent performance advantage of younger adults on tasks that assess more primary numerical abilities. The latter prediction is based on the assumption of a more or less general decline in processing speed across domains (Myerson et al, 1990; Salthouse, 1996) and does not necessarily follow a priori from an evolutionary perspective, as noted below.

BIOLOGICALLY PRIMARY AND BIOLOGICALLY SECONDARY FORMS OF COGNITION

The first subsection provides a general definition of primary and secondary abilities, and the second presents a discussion of how this demarcation might be used in cognitive aging research. The later *Numerical*

Cognition section presents a description of primary and secondary numerical abilities and issues associated with their assessment in the present study.

Primary and Secondary Abilities

Primary abilities are defined as cognitive competencies that emerge in the context of children's natural activities, such as play; are found in schooled and unschooled populations; show homologous (shared ancestor) or analogous (adaptation to similar environmental demands, without a shared ancestor) features in related species; and served a plausible evolutionary function related to growth, reproduction, or survival (Anderson & Schooler, 1991; Darwin, 1859, 1871; Eibl-Eibesfeldt, 1989; Pinker & Bloom, 1990; Shepard, 1994).[1] *Secondary abilities* are defined as cognitive competencies that emerge in unnatural contexts, such as school, and as a result of engaging in relatively unnatural activities. Many of the competencies that are learned in school such as reading and complex arithmetic, do not emerge in unschooled populations and typically do not emerge in the absence of intentional and organized activities that are designed to foster their acquisition; they are therefore considered to be biologically secondary (Geary, 1995).

Cognitive Aging Research

This section presents a brief sketch of the ways in which the evolutionary perspective on human cognition might inform cognitive aging research and provides the general rationale for the primary–secondary distinction used in the current study.

First, it should not be assumed that an evolutionary approach necessarily predicts peak cognitive performance during the early reproductive years and a uniform gradual decline thereafter (Birren & Fisher, 1992). Selection pressures can affect longevity and presumably the pattern of cognitive change in adulthood long after the onset of the reproductive years and even past the age of reproduction, if long-lived individuals contribute to the

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[1]We conceptualize both primary and secondary forms of cognition as hierarchically organized into domains, abilities, and competencies. Domains, such as language or arithmetic, represent constellations of more specialized abilities, such as language comprehension or counting. Individual abilities, in turn, appear to be comprised as goal structures (implicitly knowing why the ability is used; e.g., language is used to influence social interactions) and three types of competencies, procedural skills (knowing what to do), conceptual knowledge (knowing how to do it), and utilization knowledge (knowing when and where to do it). See Geary (1996b) and Siegler and Crowley (1994) for a discussion.

survival of their adult children and their grandchildren, that is, selection pressures would favor individuals who invested in kin throughout their life span (Hamilton, 1975). Similarly, in preindustrial societies there are often age-related changes in the division of labor that might reflect selection for age-dependent change in cognitive competencies. With male individuals, for instance, political influence tends to increase from early adulthood through middle age and even later, but, during this same time span, hunting skills decline considerably (e.g., Hill & Hurtado, 1996). Given that both hunting and political prowess influence reproductive success in men (Symons, 1979)—indicating that they have been important influences on human evolution (e.g., Hill, 1982)—this pattern suggests that selection pressures might have favored the maintenance of some cognitive competencies more than others with adult aging. With this example, selection pressures would favor men who maintained a relatively high level of those verbal and social skills that facilitate the acquisition and maintenance of political power well past young adulthood (e.g., Pinker & Bloom, 1990). Although it is speculation on our part, it is possible that the earlier described pattern of age-related change in lexical and nonlexical domains reflects such age-dependent selection pressures. Either way, this view suggests a rather complicated pattern of age-dependent change in cognitive performance during the adult years.

Moreover, it should be noted that (a) primary and secondary abilities refer to domains of cognition (e.g., Geary, 1998) and should not be confused with primary (innate maturational change) and secondary (environmental effects) aging (Anstey, Stankov, & Lord, 1993) and (b) there is probably not a one-to-one correspondence between primary–secondary abilities and other dichotomies that have been proposed in the cognitive literature, such as fluid–crystallized intelligence (Horn & Cattell, 1966), cognitive mechanics–pragmatics (Baltes, 1993), implicit–explicit learning (Schacter, 1994), automatic–effortful processing (Hasher & Zacks, 1979), or lexical–nonlexical domains (Lima et al., 1991), although there certainly are some similarities. Both primary and secondary domains, for instance, probably involve both implicit and explicit learning and knowledge. The initial development of primary abilities appears to be guided by innate implicit knowledge of the domain that directs the attentional systems of the infant to specific features of the environment and structures the processing of this information. In contrast, implicit knowledge associated with secondary abilities (e.g., reading, typing) and the automatic execution of the component processes associated with these abilities develop only after exposure to the content of the domain. Thus, whereas both primary and secondary abilities include implicit knowledge and the automatic processing of content-specific information, implicit knowledge provides the foundation for the development of primary abilities but is simply one outcome of the development of secondary abilities.

Another important clarification concerns the likely mix of primary and secondary competencies in many of the just noted domains. For instance, language is almost certainly a primary domain (Pinker & Bloom, 1990) and provides the foundation for many lexical tasks. Natural language, however, is a form of cognition that has been designed by selection pressures to regulate social interaction, among other things. Lexical tasks in experimental psychology tap some features of the language domain, such as word production, but also require secondary competencies, such as reading. Moreover, the content of different lexical tasks probably vary in the extent to which primary (animate vs. inanimate categorization) and secondary (alphabet case judgment) knowledge is assessed. Similarly, nonlexical tasks also appear to span different primary (e.g., episodic spatial memory; Uttl & Graf, 1993) and secondary (e.g., memory scanning) domains.

These issues indicate that the application of evolutionary models to issues of cognitive aging requires a careful analysis of the nature and source of the competencies underlying task performance. Any such analysis requires a consideration of the pattern of domain-relevant competencies across cultures, and sometimes across species, along with a consideration of how unnatural activities, such as schooling, might affect the competencies in question. The next section provides an example of such an analysis for the numerical and arithmetical domains and the conceptual basis for the current study.

NUMERICAL COGNITION

On the basis of the pattern of numerical and arithmetical competencies across species and cultures and the pattern of developmental change in schooled and unschooled populations, Geary (1994, 1995, 1996b) argued that a set of biologically primary numerical and arithmetical abilities can be distinguished from a larger set of biologically secondary numerical, arithmetical, and mathematical abilities. For an empirical investigation of age-related differences in speed of processing, the task involves identifying experimental procedures that assess specific primary and secondary competencies and enable a componential analysis of the speed of executing the underlying processes.

Of course, all secondary abilities are supported at some level by more primary systems, and the experimental measurement of primary abilities is dependent to some extent on secondary skills, such as using a computer (Geary, 1995, 1996b). Nevertheless, it is reasonable to assume that tasks vary in the extent to which they tap primary and secondary abilities and, given this, carefully chosen tasks should enable an assessment of age-related differences in the speed and accuracy of executing primary and secondary processes. For the current study, we chose four tasks that appear to vary in the extent to which they draw on primary and secondary competencies: enumeration, magnitude comparison, and simple and complex subtraction.

Enumeration

Considerable evidence supports the view that human infants and animals from many other species are able to enumerate or quantify the number of objects in sets of three to four items (Antell & Keating, 1983; Boysen & Berntson, 1989; Pepperberg, 1987; Starkey, 1992; Wynn, 1996). This process that is termed *subitizing* (Klahr & Wallace, 1973) and is defined as the ability to quickly and automatically quantify small sets of items without counting. These comparative and developmental patterns, among other things, strongly support the position that subitizing is a primary numerical ability (Davis & Pérusse, 1988). For the enumeration of larger set sizes, adults typically count the items or guess (Mandler & Shebo, 1982). Although there is evidence for the existence of a primary preverbal counting mechanism in human infants and other animals (e.g., Boysen & Berntson, 1989; Starkey, 1992), verbal counting in adults likely reflects a combination of primary and secondary competencies. Primary features include an implicit understanding of counting and secondary features include learning the quantities associated with numbers beyond the subitizing range. Thus, subitizing appears to represent a more pure primary enumeration process than does counting.

Several recent studies have examined adult age differences in the speed of subitizing and counting, with mixed results (Kotary & Hoyer, 1995; Nebes, Brady, & Reynolds, 1992; Sliwinski, 1997; Trick, Enns, & Brodeur, 1996). Sliwinski found that speed of subitizing decreased linearly across 20- to 80-year-old adults but that speed of counting did not differ across this age range. Kotary and Hoyer and Nebes et al. also found slower subitizing speeds for older than younger adults, but the difference was not statistically significant in the latter study. In contrast, Trick et al. found no difference in the speed of subitizing comparing 22-year-olds and 72-year-olds but an advantage for younger adults in speed of counting. The issue of adult age differences in speed of subitizing and counting is obviously not yet resolved.

Magnitude Comparison

For adults, the speed of determining which of two numbers is smaller or larger becomes slower as the magnitude of the numbers increases (e.g., 3 1 vs. 9 7) but becomes faster and less error prone as the distance between the two numbers increases (e.g., 7 9 vs. 1 9; Moyer & Landauer, 1967). It has been argued that these RT and error patterns reflect the nature of the underlying magnitude representations, on the assumption that number comparisons involve mapping each number onto the associated magnitude representation and then comparing these representations. Representations for smaller quantities appear to be more sharply defined than the representations for larger quantities, making their access quicker. The boundaries for adjacent quantities are assumed to be fuzzy, which increases error rates when comparing adjacent numbers, such as 7 8 (Gallistel & Gelman, 1992). The issue for the current study is which magnitudes associated with the numbers 1 to 9, inclusive, are primary and which are secondary. Although this issue is currently debated (Gallistel & Gelman, 1992; Geary, 1995), the most conservative approach is to assume that the limit of these innate representations is quantities associated with 1 to 3, inclusive.

Subtraction

It appears that human infants, preverbal children, and even the common chimpanzee (*Pan troglodytes*) are able to add and subtract items from sets of up to three (sometimes four) items (Boysen & Berntson, 1989; Simon, Hespos, & Rochat, 1995; Starkey, 1992; Wynn, 1992). Although this primary knowledge almost certainly provides the initial framework for the school-based learning of simple addition and subtraction, most of the formal arithmetic skills learned in school appear to be biologically secondary (Geary, 1994). For instance, the simple counting and arithmetic skills that often emerge in unschooled populations differ considerably from the base-10 arithmetic that is taught in modern schools (e.g., Saxe, 1982). The issue for the current study is whether arithmetic fact retrieval and borrowing represent primary or secondary processes.

Effective borrowing is dependent on a conceptual understanding of the base-10 structure of the Arabic number system and on school-taught procedures (Geary, 1994). On this basis borrowing is classified as a secondary competency. Arithmetic fact retrieval is less readily classified, however. Certainly the content (i.e., the arithmetic tables) is school-taught, but the memory processes supporting this content are likely to be more primary, as noted earlier. Stated otherwise, we are confident that the processes that directly enable borrowing are secondary, but we are less certain about arithmetic fact retrieval, given that the associated processes engage primary language systems but are also affected by formal schooling (Geary, Bow-Thomas, et al., 1996).

PRESENT STUDY

The methods used in the current study included—for three of the four tasks—trial-by-trial assessments of the strategies used in problem solving and enabled the measurement of the speed and accuracy of executing an array of numerical and arithmetical processes. Included among these processes are arithmetic fact retrieval, borrowing, subitizing, magnitude comparison within and beyond the subitizing range, counting, and more basic, and presumably primary, processes such as encoding and speaking numbers. Of these, age-related differences for the speed and accuracy of subitizing and borrowing are of the greatest theoretical interest. This is because the associated variables appear to represent the purest assessment of primary and secondary processes available in this array.

The method thus allows for a test of the prediction, on the basis of general slowing models, that younger adults execute nearly all cognitive processes faster than older adults and a determination of the degree of slowing for lexical (i.e., retrieval) and nonlexical (i.e., borrowing) processes. In the absence of general slowing, the method enables a determination of whether the pattern of age-related differences in speed of processing can be understood in terms of the primary–secondary distinction and thus provides a test of our earlier described hypothesis.

METHOD

Participants

The participants included 24 younger adults (11 men, 13 women) and 28 older adults (8 men; 20 women), none of whom had participated in any of our previous cognitive arithmetic studies. The younger adults were undergraduate students who received course credit for participating in this study, whereas the older adults were recruited from two retirement community centers in Columbia, Missouri, and were paid a small fee for their participation.

At the beginning of the experiment, all participants were asked to first respond to survey items on age, level of education, and self-evaluated health status. The health status item was recorded using a 1 (excellent) to 5 (bad) Likert-type scale. In between the administration of the cognitive subtraction tasks and the set of enumeration and magnitude comparison tasks, the participants were given the second half of the Wechsler Adult Intelligence Scale–Revised (WAIS-R; Wechsler, 1981) Vocabulary test. The descriptive information for the survey questions and the Vocabulary test is shown in Table 1. An analysis of variance (ANOVA) confirmed an advantage of the older adults for years of education, $F(1, 50) = 22.48$, $p < .0001$, $MSE = 3.8$, and vocabulary, $F(1, 50) = 5.67$, $p < .05$, $MSE = 30.5$, but not for health status, $F(1, 50) = 1.52$, $p > .10$, $MSE = 0.56$.

Table 1. Descriptive information for participant background

	Younger		Older	
Variable	***M***	***SD***	***M***	***D***
Age	19.0	1.2	73.4	8.8
Health status	1.7	0.5	2.0	0.9
Years of education	12.1	0.4	14.6	2.6
WAIS-R vocabulary	18.4	5.7	22.0	5.4
Gender (% female)	54		71	

Note. WAIS-R = Wechsler Adult Intelligence Scale–Revised.

Experimental Tasks

Enumeration

The stimuli consisted of 21 sets of 1 to 7 Xs, inclusive; each set size was presented 3 times. Each set of Xs was presented in a randomly determined location within a $21 \times 13\ cm^2$ area. For each set, the location of the Xs was the same for each participant, but the order of stimuli presentation differed randomly across participants, with the constraint that the same set size was not presented across consecutive trials.

Magnitude Comparison

The stimuli consisted of 84 pairs of single-digit integers ranging in value from 1 to 9, inclusive, but excluding tie pairs (e.g., 3 3, 9 9). Three sets of stimuli comprised the 84 pairs. The first set consisted of the 6 items derived from the pairwise combination of the integers 1 to 3, inclusive; each combination was presented twice, yielding 12 items. The second set consisted of the 30 items derived from the pairwise combination of the integers 4 to 9, inclusive, and the third set included 42 combinations of one integer taken from each of the first two sets (e.g., 35; 92). For half of the items, the participants were required to determine the larger number, and for the remaining items, the smaller number. The items were presented to all participants using the same randomly determined order, with the constraint that no numbers were repeated across consecutive trials. The first (smaller quantities) and second (larger quantities) sets consisted of theoretically important items, as the associated magnitudes are within and beyond the subitizing range, respectively. The set of mixed items (e.g., 3 8) was included to enable evaluation of the standard distance effects and to avoid the possibility of participants becoming aware of the discrete categories of smaller and larger combinations.

Simple Subtraction

The simple subtraction stimuli were identical to those used in Geary et al. (1993) and consisted of the 36 pairs of minuends (the top number) and subtrahends (the bottom number) that are defined by the pairwise combination of the integers 1 to 9, inclusive, that produce a positive difference (e.g., 9–3). Across problems and position, each integer appeared eight times. All participants received the same randomly determined order of problem presentation, with the constraint that no integer was repeated in the same position across consecutive trials.

Complex Subtraction

The 56 complex subtraction stimuli were also identical to those used in Geary et al. (1993). Each problem consisted of a two-digit minuend and a single-digit subtrahend, excluding 1 and 0 (e.g., 93–5). Across problems, each of the integers, 2 to 9, served as subtrahends seven times, but all minuends were unique. In addition, half of the problems required the borrow operation (e.g., 53–9), but the other half did not (e.g., 67–4). All participants received the same randomly determined order of problem presentation, with the constraint that no integer was repeated in the same position across consecutive trials.

Apparatus

The stimuli for all four tasks were presented on a $24 \times 18\ cm^2$ video screen controlled by a personal computer. A Cognition Testing Station clocking mechanism ensured the collection of RTs with an accuracy of about ±1 ms. The timing mechanism was initiated with the presentation of the stimulus on the video screen and was terminated when the participant spoke the answer into a voice-operated relay interfaced with the computer. For each item, a READY prompt appeared at the center of the video screen for 1,000 ms, followed by a blank screen for 1,000 ms. Then an item appeared on the screen and remained until the participant responded. The experimenter initiated each item presentation sequence with a control key.

Procedure

All participants were tested individually and in a quiet room. Half of the participants were first administered the simple and complex subtraction tasks and then the set of enumeration and magnitude comparison tasks. The other half were first administered the set of enumeration and magnitude comparison tasks followed by the subtraction tasks. In all, there were eight possible orders of task presentation, and participants were randomly assigned to one of these orders.

For each of the four tasks, the spoken answer was recorded on a trial-by-trial basis. For trials on which the voice operated relay was triggered (e.g., by a cough) before the answer was spoken, the RT for that trial was noted as spoiled and was not used in any subsequent analyses, although, if available, the strategy data were used.[2] For the simple and complex subtraction tasks and the enumeration task, participants were also asked, after responding to each individual item, how they arrived at the answer. Studies of verbal reports of strategy usage in arithmetic indicate that the reports were typically veridical, if the participant is asked to describe the associated procedures after each and every trial (e.g., Geary & Wiley, 1991; LeFevre et al., 1996; Siegler, 1989).

The participants described two strategy approaches to solving simple subtraction problems. The most frequently described process was "simply remembered" the answer, which was coded as retrieval. Occasionally, a participant described using the addition reference strategy. Here, the subtraction problem was solved by reference to the complimetary addition problem. For instance, one participant described solving the problem 9 —4 through reference to 5 + 4 = 9 (Geary et al., 1993; Siegler, 1989).

For solving complex subtraction problems, four different strategic approaches were reported (see Geary et al., 1993). First, a few participants reported counting down to get the answer. Counting down involved stating the value of the minuend and then counting backwards the number of times indicated by the value of the subtrahend; for example, counting "twenty-four, twenty-three, twenty-two, twenty-one," to solve 24 —3. Decomposition was used by some participants and involved breaking the subtrahend into two smaller numbers, which, in turn, were directly subtracted from the minuend. For instance, to solve the problem 35 —9, a few participants used the following sequence: 35 —5 = 30, 30 —4 = 26. Complex subtraction problems were occasionally solved by means of a rule. The rule typically involved increasing the value of the subtrahend to 10, subtracting 10 from the minuend, and finally adding the difference between 10 and the initial subtrahend to the provisional answer. For instance, to solve 35 —9, the following sequence was used by a few participants: 35 —10 = 25; 10 —9 = 1; 25 + 1 = 26. The most frequently used strategy was columnar retrieval. To solve 35 —9, the first process involved borrowing 10 from the tens value of the minuend (i.e., 30). The resulting difference (i.e., 20) was held in working memory, whereas the 10 was added to the units value of the minuend

[2]For the enumeration task, 7% and 10% of the trials were spoiled for the younger and older groups, respectively. For the simple subtraction task, 4% and 3% of the trials were spoiled for the younger and older groups, respectively; the respective percentage of spoiled trials for complex subtraction was 14 and 15. There were no spoiled trials for either group for the magnitude comparison task.

(i.e., 5). The final processes involved 15 —9 = 6, and then adding this difference to 20 to get the final answer of 26.

During the enumeration task, participants described three different ways of quantifying the arrays of Xs; subitizing, grouping, and counting. The strategy was coded as subitizing when the participants indicated that they "simply knew the number." Grouping involved subitizing subsets of Xs and then adding the associated magnitudes together. For instance, for sets of five Xs, many participants described adding sets of two and three items. Finally, participants sometimes directly counted the Xs.

RESULTS

For ease of discussion, the results are reported in four sections. The first two present group-level results for the enumeration and magnitude comparison tasks, respectively, whereas the third section presents group-level results for the two subtraction tasks. The final section presents results from a componential analysis of RTs from each of the experimental tasks. Unless otherwise noted, effects are considered significant if $p < .05$.

Enumeration Task

The analyses of the enumeration data were done separately for three set sizes: 1 to 3 items, 4 items, and 5 to 7 items. As noted earlier, people often subitize to quantify sets of 1 to 3 items but count to determine the numerosity of sets of 5 or more items (Mandler & Shebo, 1982). For sets of 4 items, the processes used to determine numerosity appear to vary between subitizing and counting. Table 2 shows the frequency of the reported strategic approaches to the enumeration task, along with the associated error rates and RTs across these three sets; mean RTs and standard deviations for each set size, averaged across strategies, are presented in Appendix A.

Sets of 1 to 3 Items

As shown in Table 2, subitizing was almost always reported to have been used by both the younger and older participants to apprehend the numerosity of sets of 1 to 3 items, although the group difference for the frequency of using subitizing was significant, $F(1, 50) = 7.02$, $MSE = 0.006$. However, this significant effect needs to be interpreted with some caution, given the ceiling effect for the younger participants. There were very few errors for this range of items for either group, and the small advantage of the younger adults was not significant, $F(1, 50) = 2.45$, $MSE = 0.007$, although their advantage for overall mean RTs was significant, $F(1, 50) = 27.04$, $MSE = 30{,}788$.

Table 2. Characteristics of enumeration task

	Strategy usage				(%) Errors				(%) RT (ms)			
	Younger		Older		Younger		Older		Younger		Older	
Strategy	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
1–3 items												
Subitizing	100	0	94	23	0	0	3	18	656	181	899	291
Grouping	—	—	2	13	—	—	0	0	—	—	1,529	745
Counting	—	—	4	20	—	—	10	32	—	—	1,059	276
4 items												
Subitizing	54	50	67	47	0	0	4	19	823	288	915	236
Grouping	38	49	12	33	0	0	0	0	909	189	1,054	501
Counting	8	28	21	41	0	0	28	46	1,038	293	1,185	398
5–7 items												
Subitizing	4	21	22	41	0	0	11	31	1,041	295	1,218	346
Grouping	73	45	37	48	8	27	7	25	1,375	445	1,415	477
Counting	23	42	42	49	4	20	15	36	1,927	546	1,835	727

Note. Reaction times (RTs) are based on correct trials. RTs that were ±2 standard deviations from the mean RT—calculated for each individual, set size, and strategy—were deleted and replaced by the appropriate mean. The 2% of RTs thus replaced likely represent strategy misclassifications.

Finally, regressing individual RTs on the number of items in the set, described in the *Componential Analysis* section, yielded mean regression coefficients that were too low to be consistent with the use of counting to quantifying sets of 1 to 3 items (Landauer, 1962). On the basis of this finding and the low overall subitizing RTs, it would appear that the participants' description of how they apprehended these small quantities was veridical.

Sets of 4 Items

There was considerable variability in the reported strategic approaches to quantifying sets of 4 items for both the younger and older adults, as shown in Table 2. Basically, the number of sets quantified by means of grouping and counting became more substantial in comparison to the sets of 1 to 3 items.[3] The younger adults reported using the grouping method more frequently than the older adults, $F(1, 50) = 9.28$, $MSE = 0.010$, but

[3]Examination of individual protocols indicated that for the enumeration of sets of 4 items, 19 of the 28 older participants always used counting; two, three, and four of the remaining older participants used grouping, subitizing, and counting, respectively, on 2 of the 3 items. Eleven of the 24 younger adults always counted, while 5, 5, and 1 participant used grouping, subitizing, and counting, respectively, on 2 of the 3 items. The last two younger participants used each strategy once.

there were no group differences for reported use of subitizing, $F(1, 50) = 1.23$, $MSE = 0.018$, or counting, $F(1, 50) = 2.56$, $MSE = 0.010$. A 2 (group: younger and older) ×3 (strategy: subitizing, grouping, and counting) mixed ANOVA for error rate, with group as a between-subjects factor and strategy as a within-subjects factor, yielded a significant group effect, favoring the younger adults, $F(1, 70) = 5.78$, $MSE = 0.03$, but nonsignificant strategy, $F(2, 70) = 2.78$, $MSE = 0.03$, and interaction, $F(2, 70) = 2.20$, $MSE = 0.03$, effects.

The same overall pattern of mean RTs was evident for the younger and older adults: subitizing RTs < grouping RTs < counting RTs. A 2 (group) × 3 (strategy) mixed ANOVA on mean RTs confirmed a significant main effect for group, favoring the younger adults, $F(1, 70) = 4.58$, $MSE = 67{,}568$, but the strategy, $F(2, 70) = 2.99$, $MSE = 67{,}568$, and interaction, $F(2, 70) < 1$, effects were not significant. Thus, although the pattern of RTs across strategies was the same for the younger and older adults, the finding that these overall differences were not significant indicates that the strategic reports for enumerating sets of 4 items need to be interpreted with some caution.

Sets of 5 to 7 Items

To quantify sets of 5 to 7 items, the older adults reported using subitizing, $F(1, 50) = 6.76$, $MSE = 0.056$, and counting, $F(1, 50) = 3.86$, $MSE = 0.213$, more frequently than the younger adults, who, in turn, reported using the grouping method more frequently than the older adults, $F(1, 50) = 14.77$, $MSE = 0.114$.[4] In keeping with the findings from sets of 4 items, younger adults committed fewer errors than their older peers, but a 2 (group) ×3 (strategy) mixed ANOVA showed that this difference did not reach statistical significance, $F(1, 96) = 3.35$, $MSE = 0.08$, nor did the main effect for strategy or the interaction, $F\text{s}(2, 96) < 1$.

Across strategies, the same pattern of mean RTs emerged for the younger and older adults, and a 2 (group) ×3 (strategy) mixed ANOVA confirmed a significant RT difference across these strategies, $F(2, 96) = 18.59$, $MSE = 179{,}767$, but yielded no significant group or Group × Strategy effects, $F(1, 96) = 2.19$, $MSE = 179{,}767$, and $F(2, 96) < 1$, respectively. For both groups, mean subitizing RTs were significantly faster than mean grouping RTs, which, in turn, were significantly faster than mean counting RTs ($p\text{s} < .05$). Even though this pattern provides some support

[4]Examination of individual protocols indicated that for the enumeration of sets of 5 to 7 items, 9 of the 28 older adults reported using the same strategy to solve all of the items; 5 of them reported using subitizing, 3 reported using grouping, and 1 reported using counting. The remaining older participants reported using a mix of strategies. Three of the 24 younger adults reported using grouping on all of the trials, and 1 participant reported counting on all of the trials. The remaining younger adults reported using a mix of strategies.

for the validity of the strategy reports, the reported use of subitizing for the sets of 5 to 7 items should be interpreted with some caution, given that subitizing does not appear to occur with any substantial frequency for sets of more than 4 items (e.g., Simon & Vaishnavi, 1996).

Magnitude Comparison Task

Two sets of analyses were conducted for the magnitude comparison task. In the first, the standard distance effect, for RTs, RT standard deviations, and error rates, was assessed across the entire range of items; mean RTs and standard deviations across distance levels are presented in Appendix B. In the second, the distance effect was analyzed across the three sets of items described in the Method section, that is, smaller, mixed, and larger sets.

Distance Analysis

Figure 1 shows mean RTs, RT standard deviations, and error rates for the absolute distance between the two presented numbers for the groups of younger and older adults; errors were too infrequent to conduct a meaningful analysis. An 8 (distance: the difference between the two presented numbers) × 2 (group) mixed ANOVA confirmed that RTs decreased monotonically as the distance between the two numbers increased, $F(7, 350) = 35.09$, $MSE = 1{,}302$. The main effect for group was also significant and favored the younger adults, $F(1, 50) = 45.28$, $MSE = 44{,}814$, but the Group × Distance interaction was not, $F(7, 350) = 1.25$, $MSE = 1{,}302$. A second 8 (distance) × 2 (group) mixed ANOVA for RT standard deviations also yielded a significantly decreasing trend as the distance between the two presented numbers increased, $F(7, 350) = 8.88$, $MSE = 887$, a significant group effect, favoring the younger adults, $F(1, 50) = 18.72$, $MSE = 2{,}201$, but a nonsignificant Group × Distance interaction, $F(7, 350) = 1.92$, $MSE = 887$. In all, these analyses suggest that the younger and older adults were using similar processes during the magnitude comparisons, but the older adults were slower and more variable in the speed of executing at least a subset of these processes.

Setwise Analysis

The mean RTs for the smaller, mixed, and larger sets across the two groups are presented in Figure 2. A 3 (set: smaller, mixed, and larger) × 2 (group) mixed ANOVA for mean RTs revealed that both main effects and the interaction were significant: $F(2, 100) = 71.88$, $MSE = 430$; $F(1, 50) = 40.48$, $MSE = 18{,}248$, and $F(2, 100) = 7.93$, $MSE = 430$, for set, group, and Set × Group, respectively. Follow-up analyses of the interaction indicated that for the younger group, mean RTs were significantly slower for the larger set relative to the smaller, $F(1, 23) = 83$, $MSE = 398$, and mixed, F(1,

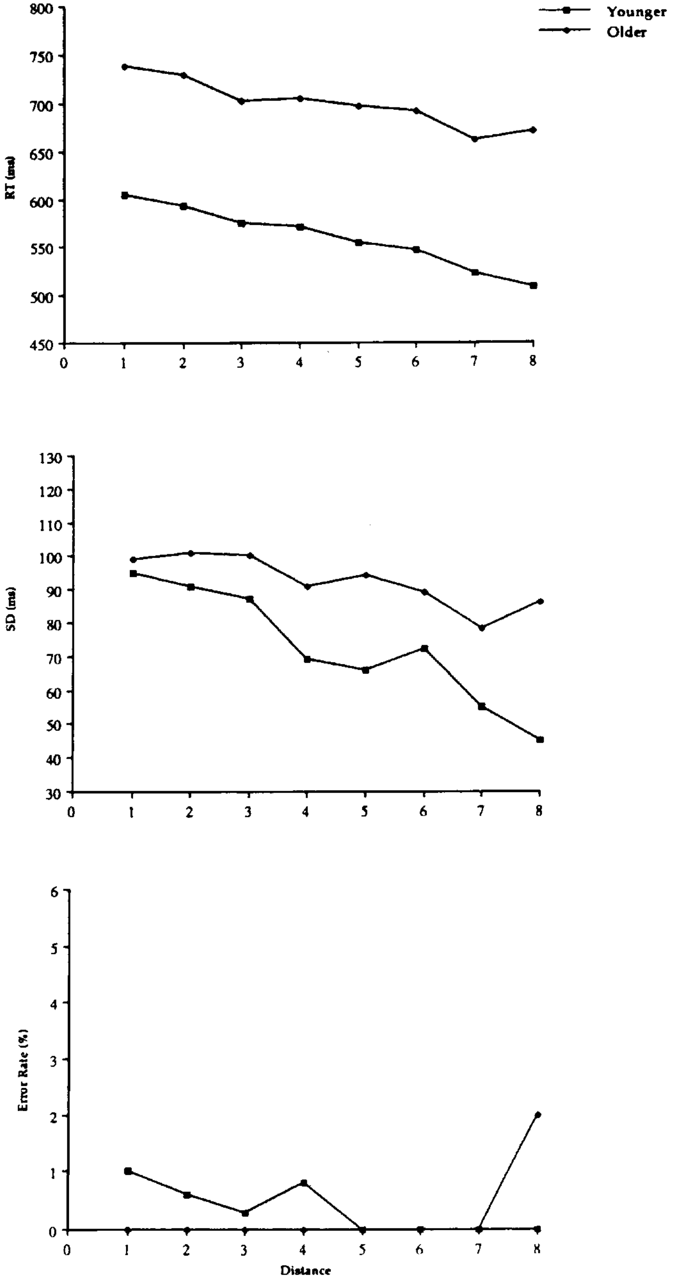

Figure 1. Mean reaction times (RTs), RT standard deviations (SDs), and error rates for the magnitude comparison task across distance level and group.

23) = 171.8, *MSE* = 229, sets, whereas mean RTs for the smaller and mixed sets did not differ, $F(1, 23) = 1.23$, $MSE = 230$. For the older group, mean RTs for the mixed set were significantly faster than mean RTs for the smaller, $F(1, 27) = 4.81$, $MSE = 638$, and larger, $F(1, 27) = 53.49$, $MSE = 330$, sets. Mean RTs for the smaller set, in turn, were significantly faster than mean RTs for the larger set, $F(1, 27) = 8.68$, $MSE = 691$.

The finding that overall RTs differed for smaller and larger set sizes for both younger and older adults suggests that the componential estimation, described later, of the speed of making magnitude comparisons should be done separately for magnitudes within and beyond the subitizing range.

Cognitive Subtraction

Simple Subtraction

Table 3 shows that both the younger and older adults reported using direct retrieval to solve nearly all of the simple subtraction problems. Error rates for direct retrieval were very low and did not differ across groups, $F(1, 50) = 2.18$, $MSE = 0.0006$, although mean retrieval RTs were significantly faster for the younger adults, $F(1, 50) = 17.58$, $MSE = 28{,}534$.

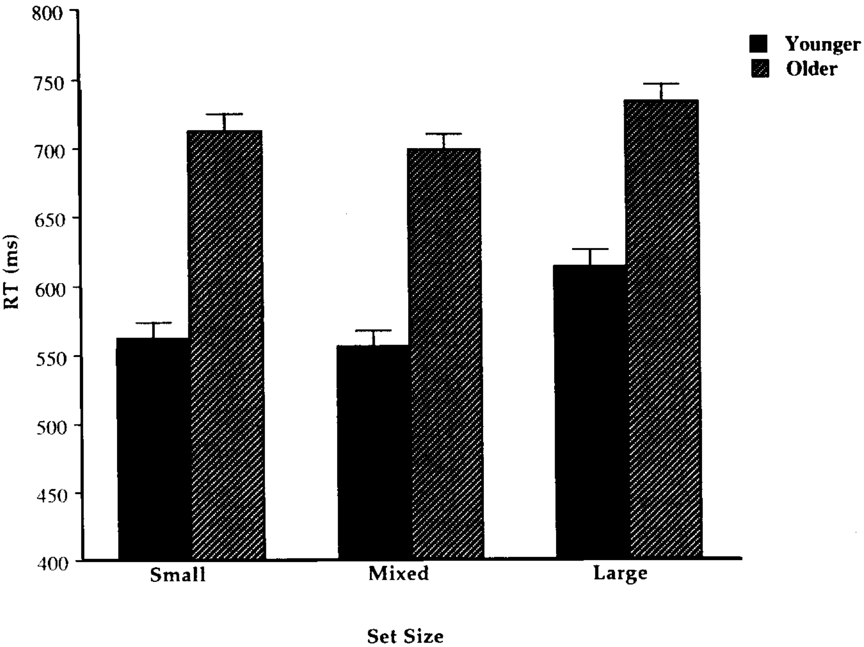

Figure 2. Mean reaction times (RTs) for magnitude comparisons within the smaller, mixed, and larger sets. The brackets indicate standard errors. Black bars = younger group; lined bars = older group.

Table 3. Characteristics of simple subtraction problems

	Strategy usage (%)				Errors (%)				RT (ms)			
	Younger		Older		Younger		Older		Younger		Older	
Strategy	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Addition reference	3	16	2	13	8	28	0	—	1,098	584	2,276	175
Retrieval	97	17	98	14	2	14	1	9	811	238	1,002	277

Note. Strategies that occurred on less than 2% of the trials are omitted. Across strategies, mean reaction time (RT) was 819.09 ms ($SD = 258.97$) for the younger adults and 1,025.01 ms ($SD = 323.49$) for the older adults.

Regressing mean RTs (averaged across participants) on the correct answer (i.e., the difference) revealed a strong linear relation for both the younger [RT = 737 + 22 (difference), $r^2 = .64$] and older [RT = 974 + 8 (difference), $r^2 = .53$] groups, as shown in Figure 3. This pattern is in keeping with the findings of Geary et al. (1993) and suggests that the difference variable provides a useful indicator of subtraction fact retrieval speeds. To provide a more readily interpretable retrieval speed estimate, we regressed

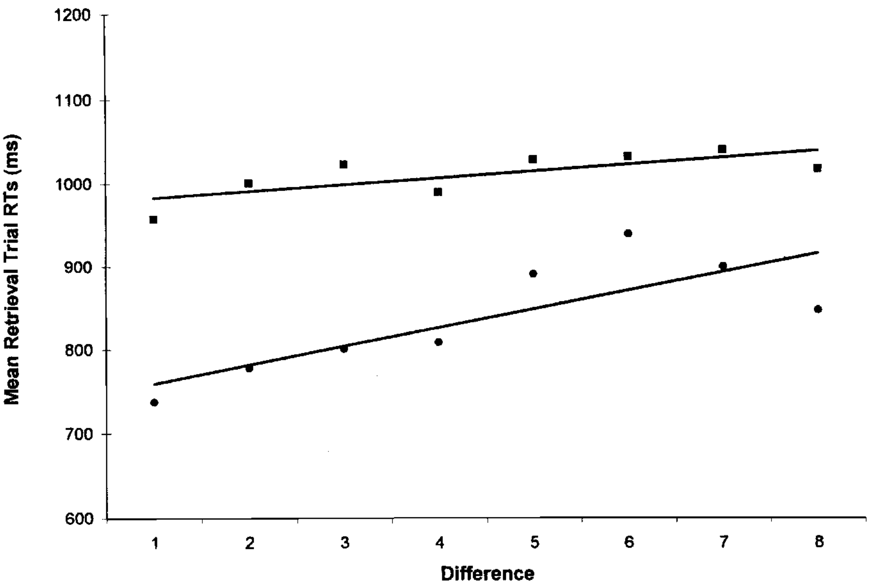

Figure 3. Mean retrieval reaction times (RTs; averaged over participants) across the problem's correct answer (i.e., difference) Square = older group; circle = younger group.

individual RTs on the difference variable—described in the *Componential Analysis* section—and obtained mean coefficients of 26 ms and 15 ms for the younger and older groups, respectively. The value of these coefficients, along with retrieval trial RTs, are again consistent with results reported by Geary et al. (1993) and are too low to reflect an implicit counting process (Landauer, 1962). These results provide support for the validity of the reported strategy choices.

Complex Subtraction

Strategy, error rate, and RT patterns are presented separately in Table 4 for complex subtraction problems that did (borrow) and did not (no borrow) require the borrow procedure. In keeping with the findings of Geary et al. (1993), the younger adults were quite variable in their strategy choices, especially for borrow problems. In contrast, the older adults consistently reported using columnar retrieval to solve complex subtraction problems whether or not the problem required the borrow operation.[5]

Table 4. Characteristics of complex subtraction problems

	Strategy usage (%)				Errors (%)				RT (ms)			
	Younger		Older		Younger		Older		Younger		Older	
Strategy	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
No borrow												
Count down	7	25	—	—	13	34	—	—	1,360	469	—	—
Columnar retrieval	93	26	99	7	6	24	2	15	1,199	336	1,409	362
Borrow												
Count down	10	30	—	—	28	45	—	—	3,847	1,252	—	—
Decomposition	40	49	—	—	12	32	—	—	2,362	1,013	—	—
Rule	4	20	—	—	14	35	—	—	2,847	675	—	—
Columnar retrieval	45	50	100	0	19	39	9	29	2,094	634	2,081	635

Note. Strategies that occurred on less than 2% of the trials are omitted. For borrow problems, mean reaction time (RT) averaged across strategies, was 2,423.15 ms (*SD* = 1,022.13) for the younger adults and 2,080.81 (*SD* = 634.81) for the older adults. For no-borrow problems, mean RT, averaged across strategies, was 1,209.99 ms (*SD* = 347.30) for the younger adults and 1,411.75 ms (*SD* = 363.90) for the older adults.

[5]Examination of individual protocols indicated that 5 of the 24 younger participants used columnar retrieval to solve all of the problems. Five of the remaining 19 participants used the columnar retrieval and decomposition strategies with roughly the same frequency. Three younger adults used columnar retrieval to solve about half of the problems and used counting and decomposition to solve the remaining problems. Two other younger adults used columnar retrieval to solve at least 60% of the problems and the decomposition and rule strategies to solve the remaining problems. The final two younger participants used some combination of all of the strategies reported in Table 4.

A 2 (group) ×2 (borrow: presence and absence) mixed ANOVA for the frequency with which columnar retrieval was used as a problem-solving strategy yielded a significant Group ×Borrow interaction, $F(1, 50) = 40.57$, $MSE = 0.009$, as well as significant effects for group, $F(1, 50) = 49.82$, $MSE = 0.012$, and borrow, $F(1, 50) = 38.47$, $MSE = 0.009$. Follow-up analyses of the interaction revealed that younger adults' strategy choices varied with the presence or absence of the borrow procedure, $F(1, 23) = 33.94$, $MSE = 0.020$, whereas older adults' strategy choices did not, $F(1, 27) = 1.50$, $MSE = 0.0001$. Basically, the group difference in the pattern of strategy choices reflects the tendency of younger adults to avoid the columnar retrieval strategy when the problem requires borrowing (Geary et al., 1993).

The older adults not only used the columnar-retrieval strategy more frequently than the younger adults, they also committed less than 1/2 the columnar retrieval errors than their younger peers, although this difference did not reach conventional significance levels, $F(1, 47) = 3.92$, $p < .06$, $MSE = 0.009$. A final 2 (group) ×2 (borrow) mixed ANOVA indicated that columnar retrieval RTs were significantly slower for borrow than no-borrow problems, $F(1, 47) = 175.57$, $MSE = 75{,}827$, but neither the group, $F(1, 47) = 2.25$, $MSE = 196{,}504$, nor Group ×Borrow, $F(1, 47) = 1.75$, $MSE = 75{,}827$, effects were significant.

For the younger adults' solving of borrow problems, the RT pattern across strategies was the same as that found by Geary et al. (1993): RTs were fastest for columnar retrieval, and slowest for counting, with decomposition and rule RTs in between; overall $F(3, 44) = 11.30$, $MSE = 631{,}990$. This pattern provides some support for the validity of the reported strategy choices.

Componential Analyses

Componential analyses provide a method for decomposing global RTs into more elementary component processes (Sternberg, 1977). In these analyses, variables that represent hypothesized elementary processes are used in regression equations to model RTs. The resulting raw regression coefficients provide an estimate of the speed of executing the associated processes (Geary & Widaman, 1987; Sternberg & Gardner, 1983; Widaman, Geary, Cormier, & Little, 1989). In the current study, this approach enabled a more fine-grained assessment of age-related differences in the speed of executing primary and secondary numerical processes than is possible with the analysis of global RTs alone. Across tasks, the reliability of the component scores was estimated by means of Cronbach's alpha, on the basis of the correlation between scores derived from odd and even numbered items.

Enumeration

Participants reported two distinct quantification processes for the enumeration task, subitizing and counting (Mandler & Shebo, 1982). As indicated from the group-level analyses, the sets of 1 to 3 items were quantified primarily by means of subitizing, whereas counting was often used to quantify sets of 4 to 7 items.

To obtain estimates of the speed of subitizing and counting, a process model that included subitizing and counting variables was fitted to individual RT data for participants who reported primarily using subitizing for the enumeration of sets of 1 to 3 items and counting for the enumeration of larger set sizes (n = 12 and 10 for the older and younger groups, respectively). Specifically, RT was regressed on a set size variable (coded the number of items in the set), a dummy coded variable (coded 0 for sets of 1 to 3 items and 1 for sets of 4 to 7 items), and their interaction (mean R = .88). The coefficient for the set size variable provides an estimate of subitizing speed, and this coefficient plus the coefficient for the interaction term provide an estimate of counting speed (Pedhazur, 1982). The intercept provided a combined estimate for speed of executing processes that were not captured by the subitizing and counting variables, presumably encoding and response times (Charness, 1981, 1987; Geary & Wiley, 1991; Widaman et al., 1989).

Mean regression coefficients and intercept values are shown in Table 5. For both age groups, the counting speed estimates reported in Table 5 are consistent with implicit counting (Landauer, 1962; Trick et al., 1996), and the estimate for speed of subitizing for the older participants is in the range reported for healthy older adults in previous studies, although the subitizing speed estimate for the younger adults is faster than is typically found for this age group (Nebes et al., 1992; Trick et al., 1996). Reliability estimates were an acceptable .73 and .62 for the subitizing and intercept variables, respectively, whereas the reliability estimate for the counting variable was too low (.05) to permit a meaningful analysis of group differences in counting speed. The low reliability estimate for the counting variable might be due to the use by some participants of a mixed grouping and counting strategy that was reported and classified as counting.

Table 5. Component scores for subitizing task (in ms)

	Intercept		**Subitizing (sizes 1–3)**		**Counting (sizes 4–7)**	
Group	***M***	***SD***	***M***	***SD***	***M***	***SD***
Younger	681	167	15	27	326	136
Older	905	257	40	197	424	252

Although the speed of subitizing in the older adults was estimated to be more than twice that estimated for the younger adults, the overall means did not differ significantly, $F(1, 20) < 1$, because of the large variance in the sample of older adults (Nebes et al., 1992). However, the advantage of younger adults for speed of executing the processes subsumed by the intercept term was significant, $F(1, 20) = 5.57$, $MSE = 48{,}821$.

Magnitude Comparison

As described earlier, the speed and accuracy of magnitude comparisons vary as a function of both the value of the larger number and the absolute distance between the larger and smaller numbers. The associated processes have been mathematically represented as follows (Dehaene, 1989; Gallistel & Gelman, 1992):

$$\mathrm{M} = \mathrm{Log_e}(\mathrm{L}/|\mathrm{L} - \mathrm{S}|), \quad (1)$$

where $\mathrm{Log_e}$ is the natural logarithm, L is the larger number, and S the smaller number. Equation 1 was used in a simple regression model, noted below, to represent individual magnitude comparison RTs (mean $R = .34$), in order to obtain estimates of the speed of executing the processes governing magnitude comparisons for smaller (including only 1 to 3 items) and larger (including only 4 to 9 items) quantities:

$$\mathrm{RT} = a + b_s(\mathrm{M_S}) + b_1(\mathrm{M_1}), \quad (2)$$

where the functions associated with b_s and b_1 model magnitude comparisons for smaller and larger quantities, respectively; $\mathrm{M_s}$ was coded 0 for larger quantities and $\mathrm{M_1}$ was coded 0 for smaller quantities. The values for b_s and b_1 thus provide an estimate for speed of making magnitude comparisons within and beyond the subitizing range, respectively, and a (the intercept) provides an estimate of the speed of executing all processes not captured by $\mathrm{M_s}$ and $\mathrm{M_1}$. The resulting mean regression weights and the intercept values for both groups are presented in Table 6. The reliability estimates for the speed of making magnitude comparisons for smaller and larger quantities were .20 and .27, respectively, and .89 for the intercept term.

Table 6. Component scores for magnitude comparison task (in ms)

	Intercept		**Small magnitude (1–3)**		**Large magnitude (4–9)**	
Group	***M***	***SD***	***M***	***SD***	***M***	***SD***
Younger	549	81	23	49	48	30
Older	681	103	45	61	41	26

A 2 (group) ×2 (size: smaller and larger) mixed ANOVA yielded a nonsignificant group effect, $F(1, 50) < 1$; a marginally significant size effect, $F(1, 50) = 3.17$, $p < .09$, $MSE = 880$; and a significant crossover interaction between group and size, $F(1, 50) = 6.17$, $MSE = 880$. Follow-up analyses showed that magnitude comparisons were made faster for the items within the subitizing range for the younger adults, $F(1, 23) = 12.57$, $MSE = 591$, but not the older adults, $F(1, 27) < 1$, although this pattern should be interpreted with caution, given the low reliability of the component scores. Again, the processes subsumed by the intercept term were executed faster by the younger adults than the older adults, $F(1, 50) = 25.41$, $MSE = 8{,}799$.

Cognitive Subtraction

The componential analyses were based on retrieval trials for simple subtraction and columnar retrieval trials for complex subtraction. For both problem types, simple regression equations were fitted to individual RT data to obtain estimates for speed of fact retrieval (simple subtraction; mean $r = .21$) and speed of executing the borrow procedure (complex subtraction; mean $r = .68$). For simple subtraction, the difference variable was used to model retrieval processes (Geary et al., 1993). For complex subtraction, RTs were modeled by a simple dummy coded variable, borrow (coded 1 for the presence and 0 for the absence of the borrow procedure). The reliability estimates for the difference and borrow variables were .59 and .79, respectively, and .93 and .65 for the respective intercept terms.

The mean regression weights and the respective mean intercept values for both the simple and complex subtraction problems are presented in Table 7. In keeping with findings from Geary et al. (1993), there was no

Table 7. Component scores for simple and complex subtraction (in ms)

	Simple problems				Complex problems			
	Intercept		Difference		Intercept		Borrow	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Younger	721	124	26	23	1,203	188	909	439
Older	938	235	15	24	1,394	237	696	319

Note. Fifty percent of the younger participants were excluded from the componential analysis for complex subtraction, because they reported using the columnar retrieval strategy for solving less than 15% of the borrow problems. Moreover, 1 younger participant and 3 older participants were excluded from the componential analysis of simple subtraction, because their component scores for the difference variable suggested that they were counting implicitly or using addition reference to solve the problems on which they reported to have retrieved the answers.

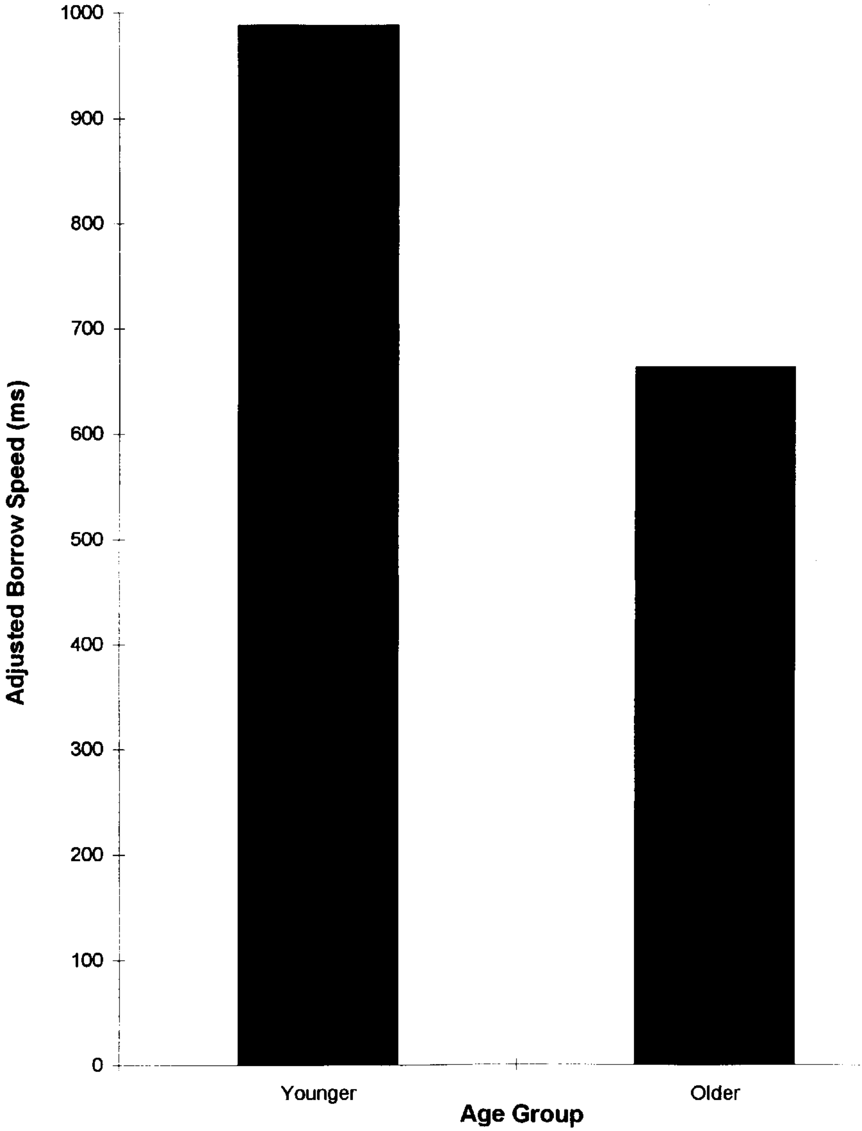

Figure 4. Estimated borrow speeds for younger and older adults, after adjusting for the group difference in years of education.

significant group difference for the difference variable, $F(1, 46) = 2.56$, $MSE = 555$, but the advantage of the older adults for speed of borrowing was marginally significant, $F(1, 40) = 3.27$, $p < .08$, $MSE = 133{,}522$. Again, the younger group showed significantly lower intercept terms for both simple, $F(1, 46) = 15.62$, $MSE = 36{,}193$, and complex, $F(1, 40) = 7.22$, $MSE = 48{,}821$, subtraction problems.

Finally, because the older adults had more years of education than the younger adults, and were therefore a potentially more select group (Salthouse & Coon, 1994), years of education was used as a covariate in a reanalysis of the group difference in borrow speed. The results are shown in Figure 4 and indicated that after controlling for years of education, the older adults' advantage for borrow speed increased from 213 ms to 325 ms and now reached conventional significance levels, $F(1, 39) = 6.31$, $MSE = 126{,}518$.[6]

Hierarchical Regression Analyses

In this section, the relation between age and component scores was analyzed by means of hierarchical regression equations, following Salthouse and Coon (1994). First, a composite intercept variable was created—on the basis of standardized scores ($M = 0$, $SD = 1$)—from the intercepts from the magnitude comparison and two subtraction tasks; the intercept for the enumeration task was not included because of the low *n*. These variables were all significantly correlated, *r*s = .69 to .77 (*p*s < .0001), and thus provide an index of general speed of executing basic numerical processes, such as encoding and speaking numbers. Next, the subitizing, magnitude comparison for smaller and larger set sizes, difference, and borrow variables were separately regressed on the composite intercept variable, years of education, health rating, vocabulary score, and age (in this order). In such analyses, nonsignificant age effects would suggest that the same factors, such as those underlying general slowing, are contributing to any age-related differences on the intercept and component process variables (Salthouse & Coon, 1994). A significant age effect, in contrast, would suggest that at least one distinct factor is contributing to the relation between age and the process variable.

First, regressing the composite intercept on years of education, health rating, vocabulary score, and age revealed a significant, $t(34) = 2.50$, age effect. The positive β coefficient (0.44) indicated that the speed of executing the processes subsumed by the intercept slowed with adult aging, above and beyond the influence of years of education, health rating, and vocabulary. Inclusion of the intercept variable with the years of education, health rating,

[6]For the older group, borrow speeds were significantly correlated with years of education, $r = .39$, such that more years of education were associated with slower borrow speeds. The reason for this pattern is not clear, but, nonetheless, suggests that the faster overall borrow speed of the older group relative to the younger group is not likely to be due to their higher mean years of education; there was little variance in the years of education of the younger participants, and thus the correlation between borrow speeds and years of education could not be computed.

and vocabulary scores revealed nonsignificant relations between age and the difference, subitizing, and large (from the magnitude comparison task) variables (all *t*s < 1.6, *p*s > .15). However, the relation between age and the small and borrow variable, $t(33) = 2.63$, $p < .01$, and $t(33) = -2.87$, $p < .01$, respectively, was significant, after controlling for the intercept variable, years of education, health rating, and vocabulary. Moreover, the β coefficient was positive for the small variable (0.53) but negative (−0.54) for the borrow variable, suggesting that the speed of executing the processes captured by the small variable were slowed above and beyond any slowing associated with the composite intercept term but that borrowing speed increased with adult aging or slowed with more recent cohorts.

DISCUSSION

The current study documented a varied pattern of age-related differences in the speed of executing numerical and arithmetical processes. Younger adults were consistently faster than older adults for executing peripheral processes, specifically, speed of encoding digits and speaking numbers (Allen et al., 1992; Charness, 1981, 1987; Geary & Wiley, 1991). However, estimates for the speed of executing more central processes sometimes favored the younger adults, sometimes favored the older adults, and sometimes showed no age-related differences. Although not all of the results were statistically significant, perhaps because of the relatively small sample sizes, the pattern of group differences and the hierarchical regression equations support the view that younger adults process numerical information within the subitizing range more quickly than older adults (Nebes et al., 1992; Sliwinski, 1997). Older adults, in contrast, were faster at borrowing in complex subtraction, whereas there were no age-related differences for the speed of subtraction fact retrieval (Allen et al., 1992; Allen et al., 1997; Geary et al., 1993; Geary & Wiley, 1991; Sliwinski et al., 1994) or for accessing magnitude representations beyond the subitizing range. Issues relating this pattern to cognitive–aging research in general are addressed in the first section below and to cognitive arithmetic in the second. The section concludes with a discussion of the implications of the primary–secondary distinction for cognitive aging research.

Cognitive Aging

The above summarized results are relevant to the issue of whether change in cognitive performance with adult aging is well captured by general slowing models or variants of these models, as with the distinction between lexical and nonlexical processing (Cerella, 1990; Lima et al., 1991; Salthouse, 1996). The results for the intercept terms, which subsume basic number encoding and production process, are certainly consistent with the position that there are changes with adult aging that slow the speed of executing some basic processes (Salthouse, 1996). However, the varied age-related pattern for speed of executing more central processes would appear to be inconsistent with strong versions of general slowing models (Cerella, 1990; Myerson et al., 1990), specifically, with the prediction that all cognitive processing is slower in older adults relative to younger adults. The finding of no age-related difference in the speed of subtraction fact retrieval, a lexical process, and an advantage of older adults for speed of borrowing, an apparently nonlexical process, would also appear to be inconsistent with predictions based on the distinction between slowing in lexical and nonlexical domains (Lima et al., 1991). The interpretation of our results is complicated, however, by apparent cohort effects in arithmetic (Schaie, 1996). It is therefore possible that speed of numerical processing does in fact decline with adult aging, with potentially different aging functions for lexical and nonlexical processes, but that cohort effects have obscured any such decline.

The real issue is how best to conceptualize exceptions to the finding that younger adults are generally faster and often more accurate than older adults on a wide variety of cognitive tasks. The pattern found in this study might be interpreted as support for the position that the speed of executing many primary cognitive processes declines with age but that the speed of executing secondary processes can show a pattern of maintenance or enhancement with adult aging. All of the processes for which younger adults were faster than older adults represent either primary competencies (i.e., accessing magnitude representations within the subitizing range) or very basic, and presumably primary, visual encoding and auditory articulation processes. Processes for which there were no age-related differences in speed of processing or an advantage of older adults all appear to be secondary competencies to some degree (Geary, 1994, 1995), that is, competencies that largely emerge with formal schooling. These processes include enumeration beyond the subitizing range (i.e., magnitude comparisons for larger numbers), arithmetic fact retrieval, and borrowing.

The results do not show that a pattern of no age-related differences or an advantage of older adults is found for all secondary abilities or that adult aging necessarily results in uniform changes in primary domains. In fact, we chose secondary tasks on which an elderly advantage might be expected on the basis of the pattern of cohort and cross-national differences in arithmetical abilities, and there is no reason to believe that this pattern necessarily extends to other secondary domains (e.g., Schaie, 1996). Regardless, the finding of an elderly advantage for speed of borrowing and the tendency of younger adults to avoid columnar retrieval when the problems required a borrow, combined with the concurrent finding of younger adults showing a faster speed of accessing primary magnitude representations, supports the central empirical hypothesis of this study—that the pattern of no age-related differences or an elderly advantage for executing arithmetical processes is most likely to be due to a schooling-related decline in arithmetical competencies in the United States (Allen et al., 1992; Geary et al., 1993; Geary et al., 1997; Geary & Wiley, 1991).

The results for complex subtraction, in particular, support this argument. This is because the columnar retrieval strategy, which was almost always used by the older adults to solve complex problems, is only learned in school. In unschooled populations, the subtraction of larger numbers (i.e., complex problems) typically involves a method that is similar to the counting down or decomposition strategies used by many of the younger adults to solve these same problems (Geary 1994; Saxe, 1982, 1985, 1988). The age difference in the strategies used for solving complex subtraction problems and in the speed and accuracy of executing the borrow procedure thus fits a pattern of more school-related experience with solving complex subtraction problems in older American adults as contrasted with younger American adults (see also Geary et al., 1997).

The finding of no age-related difference in speed of subtraction fact retrieval is consistent with previous empirical findings (Allen et al., 1992; Allen et al., 1997; Geary et al., 1993; Geary & Wiley, 1991; Sliwinski et al., 1994) but is less readily interpretable than the elderly advantage for complex subtraction. As noted in the introduction, it appears that arithmetic fact retrieval is supported by the same primary language system that supports the retrieval of words from semantic memory (e.g., Geary, 1993) but that the speed of fact retrieval is influenced by exposure to arithmetic in school (e.g., Ashcraft & Christy, 1995; Geary, 1996a), among other factors (e.g., Campbell, 1995). Thus, speed of fact retrieval is likely to be directly influenced both by any age-related change in primary semantic memory systems and by frequency of exposure to arithmetic in school. One possibility is that the finding of no difference in the speed of fact retrieval in younger and older adults reflects more exposure to arithmetic in older adults which, in turn, compensates for age-related change in speed of accessing information from semantic memory (Campbell & Charness, 1990; Charness & Campbell, 1988). Another possibility is that the speed of accessing single units of information, arithmetic facts or single words, from semantic memory is not strongly affected by the aging process (e.g., Cerella & Fozard, 1984). A testing of the limits study, where younger and older adults practice solving simple arithmetic problems—using a production task—to the point where the speed and accuracy of arithmetic fact retrieval have reached asymptotic levels is necessary to resolve this issue (Baltes & Kliegl, 1992; Campbell & Charness, 1990).

Cognitive Arithmetic

The results of the current study are also relevant to conceptual and empirical issues in cognitive arithmetic. On a conceptual level, the results suggest that cognitive models of arithmetical performance would be enhanced by the consideration of developmental, cultural, and evolutionary influences on the underlying cognitive representations and processes that support the mental solution of numerical and arithmetic problems (Geary, Hamson, Chen, Liu, & Hoard, in press). Moreover, the study suggests that the consideration of these issues in terms of primary and secondary competencies might provide a useful method for approaching the study of these influences. In essence, we are arguing that numerical and arithmetical cognition be viewed not only in terms of cognitive systems but also in terms of the contexts within which these systems have evolved and developed, that is, the study of the cognitive systems that support numerical and arithmetical cognition would be enhanced by a consideration of the functions that these systems serve in one context or another (e.g., Saxe, 1988).

On a more empirical level, the current study replicated the most provocative finding of Geary et al. (1993), that is, an advantage of older adults for speed of borrowing, and suggests that this finding was not due to the use of a relatively select sample (Salthouse & Coon, 1994). In fact, in the current study, the magnitude of the older adults' advantage in borrow speeds increased with the statistical control of years of education. We therefore conclude, at least when a production task is used to assess performance, that the advantage of older American adults over younger American adults in speed of borrowing is likely to be a real phenomenon. This finding and conclusion are in fact consistent with the hypothesis of a cross-generational decline in arithmetical abilities in the United States (Geary et al., 1997). In all, it seems likely that the conflicting results of Geary et al. (1993) and Salthouse and Coon (1994) were related to the use of production and verification tasks, respectively. The final resolution to this issue, however, requires assessing groups of younger and older adults using both tasks and items for which borrowing or carrying (e.g., as in 56×7) is required; Allen et al. (1997) found the same pattern of age differences for production and verification performance in multiplication but did not include items that required a carry.

CONCLUSION

In closing, the results of the current study are consistent with the view that the speed of information processing slows with adult aging in some fundamental cognitive systems (Salthouse, 1996), but they are inconsistent with the view that younger adults always process information more quickly than older adults (Cerella, 1990; Myerson et al., 1990). It is our position that exceptions to the pattern of declining cognitive competencies with adult aging might be fruitfully explored in terms of the primary–secondary distinction presented in this study. In particular, the distinction should be useful in determining which forms of cognition are likely to show potentially important intergenerational differences in the pattern of experiences needed for the development and maintenance of the associated cognitive competencies and which forms of cognition are not. Stated differently, the pattern of secondary abilities can vary from one culture or generation to the next; thus, there is no a priori reason to believe that younger adults always outperform older adults in these domains or vice versa. Primary competencies, in contrast, should be fundamentally the same from one generation, or culture, to the next. Given this, any decline in cognitive competencies associated with adult aging should be readily detectable in these primary domains (Baltes, 1993), although, as noted earlier, uniform decline across all primary domains does not necessarily follow from this perspective (e.g., slowing may be more pronounced in some domains than in others).

The primary–secondary distinction also provides a means of considering the extent to which experiences in childhood, such as schooling, might influence cognitive performance in old age. The result of this and other studies (e.g., Bahrick & Hall, 1991) suggest that the development of strong academic competencies, that is, secondary abilities, in childhood can have important mitigating effects on any more general declines in cognitive performance with adult aging, perhaps by facilitating their use throughout the life span. This latter implication leads us to wonder about the arithmetical competencies of the current generation of younger American adults as they age.

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APPENDIX A

Mean Reaction Time (in Milliseconds) of Enumeration Task for Each Item

	Older		Younger	
Item	***M***	***SD***	***M***	***SD***
1	937.09	214.72	651.74	145.71
2	887.65	334.85	646.97	242.92
3	920.12	363.55	670.56	136.95
4	989.33	330.22	873.19	259.97
5	1,306.69	399.72	1,203.20	425.17
6	1,403.83	588.28	1,391.13	333.60
7	1,929.49	671.98	1,860.25	559.87

APPENDIX B

Mean Reaction Time (in Miliseconds) of Magnitude Comparison Task for Each Distance Level

	Older		Younger	
Distance	***M***	***SD***	***M***	***SD***
1	738.99	127.27	603.79	119.64
2	729.87	126.70	593.71	118.10
3	701.90	129.21	574.25	111.49
4	704.95	123.71	571.40	98.35
5	696.69	125.29	554.11	98.03
6	690.60	121.74	546.29	106.64
7	661.17	119.32	522.31	90.31
8	671.24	132.00	507.02	85.17