**P9  CALCULATE AND INTERPRET CONDITIONAL PROBABILITIES THROUGH REPRESENTATION USING EXPECTED FREQUENCIES WITH TWO-WAY TABLES, TREE DIAGRAMS AND VENN DIAGRAMS (higher tier)**

**Venn diagrams**

In this section we introduce the ideas of Venn diagrams and probability. A *Venn diagram* is a way of representing information visually.

Consider two events A and B,

This section contains those numbers which are in both A **and** B

This section contains those numbers which are in A but **not** in B

This section contains those numbers which are in B but **not** in A

A

B

**EXAMPLE 1**

The Venn diagram shows information of 150 patients in a local surgery.

**EXAMPLE 1**

**EXAMPLE 1**

They were asked if they took any medication for cholesterol (C) or blood pressure (B).

**EXAMPLE 1**

(a) How many patients took both of the medications?

(b) How many patients took only one medication but not both?

(c) What does 21 represent on the Venn diagram?

(a) 57 Look at the number in the overlap of the circles

(b) 24 + 48 24 = C only as it is in circle C but not also in circle B

 = 72 48 = B only as it is in circle B but not also in circle C

(c) 21 represents the number of patients 21 is not in any of the circles

 who took no medication.

**NOTE:** If you add all the numbers you get the total number of patients.

24 + 57 + 48 + 21 = 150

In a group of 30 friends, 15 play netball, 18 play badminton, 7 play netball and badminton.

**EXAMPLE 2**

(a) Represent this information on a Venn diagram.

(b) Work out the probability that a friend plays netball only.

(c) Work out the probability that a friend plays badminton only.

(d) Work out the probability that a friend plays netball or badminton but not both.

(e) Work out the probability that a friend does not play netball or badminton.

This section contains those friends who play netball **and** badminton

This section contains those friends who play netball but **not** badminton

This section contains those friends who play badminton but **not** netball

netball

badminton

(a)

8

11

7

4

This section contains those friends who do not play either sport

Check: 8 + 7 + 11 + 4 = 30

**NOTE:** Always start from the overlap and then work your way outwards to complete the Venn diagram. Always label your circles.

(b) netball only = 15 − 7 = 8 15 play netball but 7 play both

 probability netball only =  8 play netball only out of 30 friends

(c) badminton only = 18 – 7 = 11 18 play netball but 7 play both

 probability netball only =  Write your answer as a fraction

(d) netball only or badminton only = 8 + 11 = 19 Do not include 7 as 7 play both

 probability netball only or badminton only = 

(e) not netball or badminton = 30 – (8 + 11 + 7) = 4 4 is not in any of the circles

 probability not netball or badminton = 

**EXAMPLE 3**

The following shows the results of a survey on the types of exercise taken by a group
of 100 people.

65 run, 48 swim, 60 cycle,
40 run and swim, 30 swim and cycle, 35 run and cycle
25 do all three

(a) Draw a Venn diagram to represent this information.

(b) Work out the probability that a randomly selected person from the survey

 (i) takes none of these types of exercise (ii) swims but does not run

**NOTE:** Always start from the overlap and then work your way outwards to complete the Venn diagram. In this case all of them overlap. We need to draw three overlapping circles.

Always label your circles.

R

S

C

1. All three overlap

25

2. 35 run and cycle but we already have 25 which run and cycle so another 10 run and cycle

10

3. 30 swim and cycle but we already have 25 which swim and cycle so another 5 swim and cycle

5

4. 40 run and swim but we already have 25 which run and swim so another 15 run and swim

15

5. 60 cycle but we already have 10, 25 and 5 which cycle so another 20 cycle

**NOTE** The circle which represents cycle adds up to 60

20

6. 48 swim but we already have 15, 25 and 5 which swim so another 3 swim **NOTE** The circle which represents swim adds up to 48

3

7. 65 run but we already have 15, 25 and 10 which run so another 15 run

**NOTE** The circle which represents run adds up to 65

15

8. This section represents a person who does not run, swim or cycle so

100 – (15+15+25+10+3+5+20) = 7

7

(a)

(b) (i) none of these types of exercise = 7 7 is not in any of the circles

 probability none of these types of exercise =  There are 100 people in total

 (ii) swims but does not run = 3 + 5 = 8 In circle S but not in circle R

 probability swims but does not run = 

**EXERCISE 1:**

1. Sandeep gathered some information about the pet dogs and pet cats in his road.

 There are 100 families in this road.

 68 families have a dog, 42 have a cat and 15 have a dog and a cat.

 No family had more than 1 cat or more than 1 dog.

(a) Draw a Venn diagram to represent this information.

(b) Find the probability of a family, chosen at random, having neither dog nor a cat.

2. A garage has 50 cars for sale.

 15 of the cars have air conditioning and ABS brakes

 31 of the cars have air conditioning

 17 of the cars have ABS brakes

Work out the probability of the cars that do **not** have air conditioning or ABS brakes.

3. A running club has 120 members.

 88 of the members take part in road races

 55 of the members take part in marathons

 17 of the members do not run in road races or in marathons

Work out the probability that a member only takes part in a road race or in a marathon but not both.

4. Anjali asked 60 students in her year group about where they had eaten out in the last month.

 Here are her results:

 26 had eaten in Subfood

 11 had eaten in Macdinner and Subfood

 12 had not eaten at Macdinner or Subfood

(a) Draw a Venn diagram to represent this information.

(b) Find the probability of a student who had eaten at Macdinner.

5. 90 people in a sports club were surveyed.

 19 play tennis and squash

 50 play tennis

 32 play squash

(a) Draw a Venn diagram to represent this information.

One person is chosen at random.

(b) Work out the probability that

 (i) the person chosen does not play tennis

 (ii) the person chosen plays tennis or squash or both.

6. All the members of a group of 35 students belong to at least one club.
 There are 3 clubs: chess, drama and art.

 8 of the students belong to only the art club

 6 of the students belong to all 3 clubs

 3 of the students belong to the chess and art clubs but not to the drama club

18 of the students belong to the art club

 3 of the students belong only to the chess club

 4 of the students belong only to the drama club

(a) Work out the probability that a student belongs to the chess club and to the drama club but not to the art club?

(b) Work out the probability that a student belongs to the chess club.

7. In a group of 100 students

 42 study Statistics
 40 study Mathematics
 50 study Physics
 21 study Mathematics and Physics
 19 study Statistics and Physics
 17 study Statistics and Mathematics
 5 study all three

(a) Draw a Venn diagram to represent this information.

One of the students is picked at random.

(b)   Find the probability that this student studies only **one** of these subjects.

8. 140 people were asked in a tasting survey to say which, if any, of three cakes they liked.
 Here are the results.

86 people liked cake A
93 people liked cake *B*
76 people liked cake *C*
52 people liked cakes *A* and *B*
51 people liked cakes *B* and *C*
43 people liked cakes *A* and *C*
30 people liked all three cakes.

(a)   Draw a Venn diagram to show this information.

A person is chosen at random from those who took part in the survey.

(b)   Find the probability that this person

 (i) did not like any of the three cakes,

 (ii)  liked cake *A* but not cake *B*.

9. A group of 200 adults were asked which types of magazines they read.

 Their replies showed that

82 read Sports magazines
80 read Garden magazines
84 read Fashion magazines
36 read Sports magazines and read Garden magazines
31 read Sports magazines and read Fashion magazines
25 read Garden magazines and read Fashion magazines
14 read Sports magazines and read Garden magazines and read Fashion magazines

One of the adults asked is to be chosen at random.

Find the probability that this adult

(a)   reads none of these types of magazine,

(b)  reads exactly two of these types of magazine.

**Conditional probability using Venn diagrams**

**Conditional probability** is the probability of an event occurring **given** that another event has occurred.

For example,

* the probability of David studying GCSE mathematics given that he is studying GCSE physics,
* the probability that I will pay my gas bill given that I have just been paid,
* the probability that my students will turn up to class given that it is a rainy day.

The emphasis is that the probability is influenced by something that has already happened.

P(A | B) means the probability of A occurring, given that B has already occurred.

**EXAMPLE 4**

In a class of 29 girls

 18 girls play netball
 13 girls play hockey
 7 girls play both

(a) Show this information on a Venn diagram.

One of these girls is picked at random.

(b) Write down the probability that this girl plays hockey given that this girl also
 plays netball.

(a)

7

11

6

5

N

H

This is how many play both

This shows how many do not play netball or hockey so

29 – (11 + 7 + 6) = 5

13 play hockey but 7 play both so need another 6 who play hockey

18 play netball but 7 play both so need another 11 who play netball

(b) 18 girls play netball Given plays netball so look at

 the netball circle only

 7 of these the netball players also play hockey We want to know how many of these

 netball players also play hockey

 Probability plays hockey given plays netball = 

**EXAMPLE 5**

In a group of 40 children there are 19 who can swim and 16 who can ride a bike.

There are 5 children who can swim and ride a bike.

A child is selected at random.

(a) Find the probability that this child cannot swim or ride a bike.

Another child is selected at random.

(b) Given that this child can ride a bike, work out the probability that this child can swim.

5

14

11

10

S

B

This shows how many can swim and ride a bike

This shows how many cannot swim or ride a bike so 40 – (14 + 5 + 11) = 10

16 can ride a bike but 5 of these can already ride a bike so need another 11 in the circle for bike

19 can swim but 5 of these can already swim so need another 14 in the circle for swimming

(a) It is helpful to draw a Venn diagram

 to show this information.

 Answer:  10 of the 40 children cannot swim or ride a bike

**NOTE:** You do not need to cancel the answer to 

(b) 5 + 11 = 16 ride a bike Given rides a bike so look

 at the bike circle only

 5 of these bike riders also swim We want to know how many

 of these bike riders also swim

 Probability that a child can swim given rides a bike = 

**EXAMPLE 6**

Charles asked 100 people which of the films Ghost, Titanic and Shrek they have watched. Here is some information about his results.

 55 had watched Ghost.
 58 had watched Titanic.
 60 had watched Shrek.
 36 had watched Ghost and Shrek.
 35 had watched Ghost and Titanic.
 37 had watched Titanic and Shrek.
 20 had watched all three films.

(a) Draw a Venn diagram to show this information.

Charles picks, at random, one of these 100 people.

(b) Given that the person had watched Titanic, work out the probability that this person

 had also watched Ghost.

Fill in the numbers on the Venn diagram in the order shown below ... ie starting with 20 watching all 3 films.

(a)

G

T

S

1. All three overlap

20

4. 36 had watched Ghost and Shrek but we already have 20 who have watched all three so need another 16 who have watched Ghost and Shrek

16

2. 37 had watched Titanic and Shrek but we already have 20 who have watched all three so need another 17 who have watched Titanic and Shrek

17

3. 35 had watched Ghost and Titanic but we already have 20 who have watched all three so need another 15 who have watched Ghost and Titanic

15

5. 60 had watched Shrek but we already have 16, 20 and 17 who have watched Shrek so need another 7 who watch Shrek only.

**NOTE:** The circle which represents Shrek adds up to 60

7

6. 58 had watched Titanic but we already have 15, 20 and 17 who have watched Titanic so need another 7 who watch Titanic.

**NOTE:** The circle which represents Titanic adds up to 58

6

7. 55 had watched Ghost but we already have 15, 20 and 16 who have watched Ghost so need another 4 who watch Ghost.

**NOTE:** The circle which represents Ghost adds up to 55

4

8. This section represents a person who does not run, swim or cycle so

100 – (20+15+16+17+4+6+7) = 15

15

(b) Number who watched Titanic = 58 Given watched Titanic so look

 at the Titanic circle only

 Of these 58, 15 + 20 = 35 had also watched Ghost. Look at the overlap of G and T

 Probability that this person had also watched Ghost = 

**EXERCISE 2:**

1. 90 children were asked what type of bottled water they took to school.

 Their replies are as follows:

 52 took sparkling water

 36 took still water

 14 took both types of water

(a) Show this information on a Venn diagram.

(b) Given that a child takes sparkling water, find the probability that this child also takes still water.

(c) Given that a child takes still water, find the probability that this child also takes sparkling water.

2. In a group of 40 students 6 are left-handed, 18 have size 8 feet and 2 are left-handed with size 8 feet.

(a) Find the probability that a student is left-handed or has size 8 feet,

(b) Given that the student is left-handed, find the probability that a student has size 8 feet.

3. In a survey 100 people were asked whether they watched snooker or cricket when it was on TV.

20 watched neither, 75 watched snooker, 32 watched cricket.

A person is selected at random.

(a) Find the probability that this person watched both cricket and snooker.

(b) Given that this person watched snooker, work out the probability that this person watched cricket.

4. A person's blood group is determined by whether or not it contains any of 3 substances *A*, *B* and *C*.

A doctor surveyed 300 patients' blood and produced the table below.

|  |  |
| --- | --- |
| **Blood contains** | **Number of patients** |
| Only *C* | 100 |
| *A* and *C* but not *B* | 100 |
| Only *A* | 30 |
| *B* and *C* but not *A* | 25 |
| Only *B* | 12 |
| *A*, *B* and *C* | 10 |
| *A* and *B* but not *C* | 3 |

(a)  Draw a Venn diagram to show this information.

(b)  Find the probability that a randomly chosen patient's blood contains substance *C*.

Harry is one of the patients.

(c)  Given that his blood contains substance *A*, find the probability that his blood contains all 3 substances.

5. There are 180 students at a college following a general course in computing.

 Students on this course can choose to take up to three extra options.

112 take systems support

 70 take developing software

 81 take networking

 35 take developing software and systems support

 28 take networking and developing software

 40 take systems support and networking

 4 take all three extra options

(a) Draw a Venn diagram to show this information.

A student from the course is chosen at random.

(b) Find the probability that this student takes

 (i) none of the three extra options, (ii) networking only.

Students who want to become technicians take systems support and networking.

(c) Given that a randomly chosen student wants to become a technician, find the probability that this student takes all three extra options.

6. 100 people were asked which sports they watched on television.

 Here are the results.

 36 people watched cricket
 28 people watched rugby
 36 people watched football
 17 people watched both cricket and rugby
 19 people watched both cricket and football
 15 people watched both rugby and football
 10 people watched all three sports

(a)   Draw a Venn diagram to show this information.

One of the 100 people is selected at random.

(b)   Given that a person watches cricket, find the probability that this person also watches football.

(c) Given that a person watches at least one of the sports, find the probability that this person watches all three.

7. The following shows the results of a juice tasting survey of 100 people.

96 like apple juice

93 like orange juice

96 like mango juice

92 like apple juice and orange juice

91 like orange juice and mango juice

93 like apple juice and mango juice

90 like all three

(a)  Draw a Venn diagram to represent this data.

(b)  Find the probability that a randomly selected person from the survey likes

 (i)  none of the three juices, (ii)  apple juice but not orange juice.

 (c)  Given that a person from the survey likes apple juice, find the probability that the person likes mango juice.

**ANSWERS**

**Exercise 1**

1. (a) (b) 

d

c

15

53

27

5

2.

AC

ABS

15

16

2

17

 

3.

RR

M

*x*

88 – *x*

55 – *x*

17

(88 *– x*) + (55 *– x*) + *x* +17 = 120

 *x* = 40

 

4. (a) (b) 

M

S

11

22

15

12

5. (a) (b)(i)  (ii) 

T

S

19

31

13

27

6. (a) (b) 

C

10

3

4

6

3

1

8

D

A



7. (a) (b) 

S

12

11

7

5

14

16

15

M

P

20

8. (a) (b)(i)  (ii) 

A

22

21

20

30

13

21

12

B

C

1

9. (a)  (b) 

S

22

29

33

14

17

11

42

G

F

32

**Exercise 2**

1. (a)

SP

ST

14

38

22

16

(b)  (c) 

2.

LH

S8

2

4

16

18

(a)  (b) 

3.

S

C

*x*

75 – *x*

32 – *x*

20

(75 – *x*) + *x* + (32 – *x*) + 20 =100

 *x* = 27

(a)  (b) 

4. (a)

A

3

30

12

10

100

25

100

B

C

20

(a) 

(b) 

5. (a)

SS

31

41

11

4

36

24

17

DS

 N

16

(b)(i)  (ii) 

(c) 

6. (a)

C

7

10

6

10

9

5

12

R

F

41

(b)  (c) 

7. (a)

O

A

0

1

2

90

1

3

2

M

1

(b)(i)  (ii) 

(c) 