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| **UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry** |

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**SPECIFICATION REFERENCES**

N7 Calculate with roots and with integer **and fractional** indices

N8 calculate exactly with fractions and **surds** …

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

A4 simplify and manipulate algebraic expressions (including those involving surds) by collecting like terms …

A5 understand and use standard mathematical formulae; …

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …

G3 … understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G4 derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; …

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras’ theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G11 solve geometrical problems on coordinate axes

G20 know the formulae for: Pythagoras’ theorem *a*2 + *b*2 = *c*2, and the trigonometric ratios sine, cosine and tan; apply them to find angles and lengths in right-angled triangles … and in two dimensional figures

G21 know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45°, 60° and 90°; know the exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60°

**PRIOR KNOWLEDGE**

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trig formulae.

Students should recall basic angle facts.

Students should understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

**KEYWORDS**

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, Pythagoras’ Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, segment, length

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| **5a. Polygons, angles and parallel lines**  (G1, G3, G4, G6, G11) | **Teaching time**  5-7 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
* Understand ‘regular’ and ‘irregular’ as applied to polygons;
* Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle;
* Use symmetry property of an isosceles triangle to show that base angles are equal;
* Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
* Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
* Explain why the angle sum of a quadrilateral is 360°; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
* Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
* Use the angle sums of irregular polygons;
* Calculate and use the sums of the interior angles of polygons; use the sum of angles in a triangle and use the angle sum in any polygon to derive the properties of regular polygons;
* Use the sum of the exterior angles of any polygon is 360°;
* Use the sum of the interior angles of an n-sided polygon;
* Use the sum of the interior angle and the exterior angle is 180°;
* Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
* Calculate the angles of regular polygons and use these to solve problems;
* Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
* Use angle facts to demonstrate how shapes would ‘fit together’, and work out interior angles of shapes in a pattern.

**POSSIBLE SUCCESS CRITERIA**

Name all quadrilaterals that have a specific property.

Given the size of its exterior angle, how many sides does the polygon have?

What is the same, and what is different between families of polygons?

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Multi-step “angle chasing”-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.

Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.

**COMMON MISCONCEPTIONS**

Some students will think that all trapezia are isosceles, or a square is only square if ‘horizontal’, or a ‘non-horizontal’ square is called a diamond.

Pupils may believe, incorrectly, that:

* perpendicular lines have to be horizontal/vertical;
* all triangles have rotational symmetry of order 3;
* all polygons are regular.

Incorrectly identifying the ‘base angles’ (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

**NOTES**

Demonstrate that two line segments that do not meet could be perpendicular – if they are extended and they would meet at right angles.

Students must be encouraged to use geometrical language appropriately, ‘quote’ the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.

Emphasise that diagrams in examinations are seldom drawn accurately.

Use tracing paper to show which angles in parallel lines are equal.

Students must use co-interior, not supplementary, to describe paired angles inside parallel lines. (NB Supplementary angles are any angles that add to 180, not specifically those in parallel lines.)

Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.