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| **UNIT 19: Direct and indirect proportion: using statements of proportionality, reciprocal and exponential graphs, rates of change in graphs, functions, transformations of graphs** |

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**SPECIFICATION REFERENCES**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A12 recognise, sketch and interpret graphs of the reciprocal function  with *x* ≠ 0, **exponential functions *y* = *kx* for positive values of *k*** …

A13 **sketch translations and reflections of a given function**

A14 plot and interpret reciprocal graphs and **exponential graphs** …

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs) and interpret results in cases such distance–time graphs, velocity–time graphs and graphs in financial contexts (this does not include calculus)**

A21 translate simple situations or procedures into algebraic expressions or formulae; …

R7 understand and use proportion as equality of ratios

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R11 use compound units such as speed, rates of pay, unit pricing*,* density and pressure

R13 understand that *X* is inversely proportional to *Y* is equivalent to *X* is proportional to ; **construct and** interpret equations that describe direct and inverse proportion

R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

R15 **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus**

R16 set up, solve and interpret the answers in growth and decay problems …

**PRIOR KNOWLEDGE**

Students should be able to draw linear and quadratic graphs.

Students should be able to calculate the gradient of a linear function between two points.

Students should recall transformations of trigonometric functions.

Students should have knowledge of writing statements of direct proportion and forming an equation to find values.

**KEYWORDS**

Reciprocal, linear, gradient, quadratic, exponential, functions, direct, indirect, proportion, estimate, area, rate of change, distance, time, velocity, transformations, cubic, transformation, constant of proportionality

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| **19a. Reciprocal and exponential graphs; Gradient and area under graphs**  (R11, R14, R15, R16, A7, A12, A13, A14, A15) | **Teaching time**  6-8 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Recognise, sketch and interpret graphs of the reciprocal function  with *x* ≠ 0
* State the value of *x* for which the equation is not defined;
* Recognise, sketch and interpret graphs of exponential functions *y* = *kx* for positive values of *k* and integer values of *x*;
* Use calculators to explore exponential growth and decay;
* Set up, solve and interpret the answers in growth and decay problems;
* Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(*x*) + *a*, or f(*x* – *a*):
* apply to the graph of *y* = f(*x*) the transformations *y* = –f(*x*), *y* = f(–*x*) for linear, quadratic, cubic functions;
* apply to the graph of y = f(*x*) the transformations *y* = f(*x*) + *a*, *y* = f(*x* + *a*)   
  for linear, quadratic, cubic functions;
* Estimate area under a quadratic or other graph by dividing it into trapezia;
* Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
* Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
* for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
* for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
* Interpret the gradient of a linear or non-linear graph in financial contexts;
* Interpret the area under a linear or non-linear graph in real-life contexts;
* Interpret the rate of change of graphs of containers filling and emptying;
* Interpret the rate of change of unit price in price graphs.

**POSSIBLE SUCCESS CRITERIA**

Explain why you cannot find the area under a reciprocal or tan graph.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Interpreting many of these graphs in relation to their specific contexts.

**COMMON MISCONCEPTIONS**

The effects of transforming functions is often confused.

**NOTES**

Formal function notation along with inverse and composite functions will have been encountered but are topics that students may need to be reminded about.

Translations and reflections of functions are included in this specification, but not rotations or stretches.

Financial contexts could include percentage or growth rate.

When interpreting rates of change with graphs of containers filling and emptying, a steeper gradient means a faster rate of change.

When interpreting rates of change of unit price in price graphs, a steeper graph means larger unit price.