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| **UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof** |

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**SPECIFICATION REFERENCES**

N1 … use the symbols =, ≠, <, >, ≤, ≥

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions, **surds** …; **simplify surd expressions involving squares** …

N9 calculate with and interpret standard form *A* × 10*n*, where 1 ≤ *A* < 10 and *n* is an integer.

A1 use and interpret algebraic notation, including:

* + *ab* in place of *a* × *b*
  + 3*y* in place of *y* + *y* + *y* and 3 × *y*
  + *a*2 in place of *a* × *a*, *a*3 in place of *a* × *a* × *a*, *a*2*b* in place of *a* × *a* × *b*
  +  in place of *a* ÷ *b*
  + coefficients written as fractions rather than as decimals
  + brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions … by:

* collecting like terms
* multiplying a single term over a bracket
* taking out common factors
* expanding products of two … binomials
* factorising quadratic expressions of the form *x*2 + *bx* + *c*, including the difference of two squares; …
* simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A17 solve linear equations in one unknown algebraically …;

A20 **find approximate solutions to equations numerically using iteration**

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation …, solve the equation and interpret the solution

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences and simple geometric progressions (*rn* where *n* is an integer, and *r* is a rational number > 0), recognise and use other sequences **or a surd)**

A25 deduce expressions to calculate the *n*th term of linear sequences.

**PRIOR KNOWLEDGE**

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

* the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
* dealing with decimals and negatives on a calculator;
* using index laws numerically.

**KEYWORDS**

Expression, identity, equation, formula, substitute, term, ‘like’ terms, index, power, negative and fractional indices, collect, substitute, expand, bracket, factor, factorise, quadratic, linear, simplify, approximate, arithmetic, geometric, function, sequence, *n*th term, derive

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| **2a. Algebra: the basics, setting up, rearranging and solving equations**  (N1, N3, N8, A1, A2, A3, A4, A5, A6, A7, A17, A20, A21) | **Teaching time**  9-11 hours |

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

* Use algebraic notation and symbols correctly;
* Know the difference between a term, expression, equation, formula and an identity;
* Write and manipulate an expression by collecting like terms;
* Substitute positive and negative numbers into expressions such as 3*x* + 4 and 2*x*3 and then into expressions involving brackets and powers;
* Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. *l* × *w*, *v* = *u* + *at*;
* Simplify expressions by cancelling, e.g.  = 2*x*;
* Use instances of index laws for positive integer powers including when multiplying or dividing algebraic terms;
* Use instances of index laws, including use of zero, fractional and negative powers;
* Multiply a single term over a bracket and recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;
* Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to (2*x* + 3*y*)(3*x* – *y*);
* Know that squaring a linear expression is the same as expanding double brackets;
* Factorise quadratic expressions of the form *ax*2 + *bx* + *c*;
* Factorise quadratic expressions using the difference of two squares;
* Set up simple equations from word problems and derive simple formulae;
* Understand the ≠ symbol (not equal), e.g. 6*x* + 4 ≠ 3(*x* + 2), and introduce identity ≡ sign;
* Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
* Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
* Solve linear equations in one unknown, with integer or fractional coefficients;
* Set up and solve linear equations to solve to solve a problem;
* Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
* Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
* Use and substitute formulae from mathematics and other subjects, including the kinematics formulae *v* = *u* + *at*, *v*2 – *u*2 = 2*as*, and *s* = *ut* +  *at*2;
* Change the subject of a simple formula, i.e. linear one-step, such as *x* = 4*y*;
* Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
* Simple proofs and use of ≡ in “show that” style questions; know the difference between an equation and an identity;
* Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

**POSSIBLE SUCCESS CRITERIA**

Simplify 4*p* – 2*q*2 + 1 – 3*p* + 5*q*2.

Evaluate 4*x*2 – 2*x* when *x* = –5.

Simplify *z*4 × *z*3, *y*3 ÷ *y*2, (*a*7)2, .

Expand and simplify 3(*t* – 1) + 57.

Factorise 15*x*2*y* – 35*x*2*y*2.

Expand and simplify (3*x* + 2)(4*x* – 1).

Factorise 6*x*2 – 7*x* + 1.

A room is 2 m longer than it is wide. If its area is 30 m2 what is its perimeter?

Use fractions when working in algebraic situations.

Substitute positive and negative numbers into formulae.

Be aware of common scientific formulae.

Know the meaning of the ‘subject’ of a formula.

Change the subject of a formula when one step is required.

Change the subject of a formula when two steps are required.

**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

**COMMON MISCONCEPTIONS**

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

Hierarchy of operations applied in the wrong order when changing the subject of a formula.

*a*0 = 0.

3*xy* and 5*yx* are different “types of term” and cannot be “collected” when simplifying expressions.

The square and cube operations on a calculator may not be similar on all makes.

Not using brackets with negative numbers on a calculator.

Not writing down all the digits on the display.

**NOTES**

Some of this will be a reminder from Key Stage 3 and could be introduced through investigative material such as handshake, frogs etc.

Students will have encountered much of this before and you may wish to introduce solving equations using function machines.

Practise factorisation where more than one variable is involved. NB More complex quadratics are covered in a later unit.

Plenty of practice should be given for factorising, and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed. Encourage students to expand linear sequences prior to simplifying when dealing with “double brackets”.

Emphasise good use of notation.

Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.

Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

Use examples involving formulae for circles, spheres, cones and kinematics when changing the subject of a formula.

For substitution use the distance–time–speed formula, and include speed of light given in standard form.

Students should be encouraged to use their calculator effectively by using the replay and ANS/EXE functions; reinforce the use of brackets and only rounding their final answer with trial and improvement.