

Please write clearly in block capitals.	
Centre number	Candidate number
Surname	
Forename(s)ANSL	16RS
Candidate signature	

A-level MATHEMATICS

Unit Pure Core 4

Friday 17 June 2016

Afternoon

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



MPC4

Answer all questions.

Answer each question in the space provided for that question.

1 (a) Express
$$\frac{19x-3}{(1+2x)(3-4x)}$$
 in the form $\frac{A}{1+2x} + \frac{B}{3-4x}$.

[3 marks]

- (b) (i) Find the binomial expansion of $\frac{19x-3}{(1+2x)(3-4x)}$ up to and including the term in x^2 . [7 marks]
 - (ii) State the range of values of x for which this expansion is valid.

[1 mark]

QUESTION PART REFERENCE	Answer space for question 1
1a)	19x - 3 = A(3-4x) + B(1+2x)
	$x = \frac{3}{4} \rightarrow 19(\frac{3}{4}) - 3 = B(1 + 2(\frac{3}{4}))$ $5\frac{3}{4} - 3 = \frac{10}{4}B$
	45/4 = 10/4 B
	B=9/2 OR 4.5
	$\chi = -1/2 \rightarrow 19(-1/2) - 3 = A(3 - 4(-1/2))$
	$\frac{-19}{2} - 3 = 5A$
	$A = -\frac{5}{2} 0R - 2 - 5$
	19x - 3 = -2.5 + 4.5
	$(1+2\pi)(3-4\pi)$ $(1+7\pi)$ $(3-4\pi)$
	= -5 + 9
	$2(1+1x) \qquad 2(3-4x)$

QUESTION PART REFERENCE	Answer space for question 1
<i>bi</i>)	$\frac{-5(1+7x)^{-1}+9(3-4x)^{-1}}{2}$
	$\frac{-5(1+7x)^{-1}}{2} = \frac{-5(1+(-1)(1x)+(-1)(-1)(1x)^{2}}{2}$
	$\frac{1}{2} = -\frac{5}{2} \left(1 - 2x + 4x^2 \right)$
	$\frac{9(3-4\pi)^{-1} = \frac{9}{2}(3)^{-1}(1-\frac{4}{3}x)^{-1}}{2}$
	$=\frac{3}{2}\left[1+(-1)(-\frac{4}{3})^{2}+(-1)(-\frac{2}{3})^{2}\right]$
	$= \frac{3}{2} \left(1 + \frac{4}{3} x + \frac{16}{9} x^{2} \right)$
	$\frac{-S}{2}\left(1-2x+4x^{2}\right)+\frac{3}{2}\left(1+\frac{4}{3}x+\frac{16}{9}x^{2}\right)$
	$\frac{-S + S \chi - 10 \chi^{2} + 3 + 7 \chi + 8 \chi^{2}}{2}$
	$\frac{-1+7\chi-22\chi^2}{3}$
_i)	valid for 12x1 (1 OR]-4/3x (1 1x1 (1/2 1x1 (1/4)
	: 1/2 (X (1/2
	Turn over ▶



2 By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3\cos 2\theta - 5\cos \theta + 2 = 0$$

in the interval $0^{\circ} < \theta < 360^{\circ}$, giving your answers to the nearest $0.1^{\circ}.$

[5 marks]

QUESTION PART REFERENCE	Answer space for question 2
2)	$3\cos 2\theta - 5\cos \theta + 2 = 0$ $0 < \theta < 360^{\circ}$. $\cos 2\theta = 2\cos^{2}\theta - 1$
	$\left[\cos 2\theta = 2\cos^2\theta - 1\right]$
	3 (2cos20-1) - Scos 0 + 2 = 0
	$6(0)^{2}\theta - 3 - 5(0)\theta + 2 = 0$
	600120 - 50010 -1=0
	$(6\cos\theta + 1)(\cos\theta - 1) = 0$
	6(0)A+1=0 OR COIA-1=0
	$\cos \theta = -1/6 \qquad \cos \theta = 1$
	θ = 99.59406, ' Θ = 0' -1 out of 260.4059
	260.4059 ruge
	5 A
	180-80.4
	80.4
	7
	270
	D=99-6°, 260.4° (newest 0-1°)



QUESTION PART REFERENCE	Answer space for question 2
	١,



3 (a) Express
$$\frac{3+13x-6x^2}{2x-3}$$
 in the form $Ax + B + \frac{C}{2x-3}$.

[4 marks]

(b) Show that $\int_3^6 \frac{3+13x-6x^2}{2x-3} dx = p+q \ln 3$, where p and q are rational numbers.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 3
3a)	$3 + 13x - 6x^2 \rightarrow (-3x + 2)$
	$2x-3$ $2x-3 - 6x^2 + 13x + 3$
	$-6x^2+9x$
	0+4x+3
house, organization or many per many	0 + 4x + 3 $-4x - 6$
	0(+9)
	$3 + 13x - 6x^2 = -3x + 2 + 9$
	$\frac{3173x 6x}{2x-3}$ $\frac{7x-3}{2x-3}$
	A = -3, $B = 2$, $C = 9$
	11-3, 8-2, 6
6)	J ₃ 2x-3
	$\left[\frac{-3x^2 + 2x + 9 \ln(2x-3)}{2} \right]_3^6$
	$ \frac{\left(-3(6)^2+2(6)+\frac{9}{2}\ln(2(6)-3)\right)-\left(-3(3)^2+2(3)+\frac{9}{2}\ln(2(3)-3)\right)}{2} $
	$\left(-42 + \frac{9}{2} \ln 9\right) - \left(-15 + \frac{9}{2} \ln 3\right)$
	$\frac{-69 + 9 /n 3^2 - 9 /n 3}{2}$
	$-\frac{69}{2} + \frac{18}{2} \ln 3 - \frac{9}{2} \ln 3 = -\frac{69}{2} + \frac{9}{2} \ln 3$



QUESTION	Answer space for question 3
QUESTION PART REFERENCE	Amover space for question 5
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	N _s
	Δ.



4 The mass of radioactive atoms in a substance can be modelled by the equation

$$m = m_0 k^t$$

where m_0 grams is the initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

(a) (i) A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days. Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

[1 mark]

- (ii) A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.

 Calculate the initial mass of this sample, giving your answer to the nearest gram.

 [2 marks]
- (b) The half-life of a radioactive substance is the time it takes for a mass of m_0 to reduce to a mass of $\frac{1}{2}m_0$.

A sample of radioactive vanadium reduced in mass from exactly $10~{\rm grams}$ to $8.106~{\rm grams}$ in $100~{\rm days}$.

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 4
4ai)	$M = M_0 k^t$
	Mo = 24, M=12, 1=8
	12 = 24 X K 8
	1, = k ⁸
	$\frac{1}{k} = k^{8}$ $k = 8\sqrt{1/2} = 0.9170040. = 0.917004 (6dp)$
· i)	M = Mo x 0.917004 t M=1, t=60
	1 = Mo X 0.917004 60
	Me = 1 = 181.0198477
	0.91700460 = 18/g (newest g)



	9
QUESTION PART REFERENCE	Allswel space for question 4
6)	$M = M_0 k^{t}$ $M_0 = 10$, $M = 8.106$, $t = 100$ 8.106 = $10 \times k^{100}$
	$k = \frac{100}{8.106} = 0.9979023$
	M=5, Mo=10, K=0.9979023.
	S = 10 x 0.99 79023. t
	10g 0.5 = 6 log 0.9979023.
	10g 0.9979023.
	t=330.0882928.
	t= 330 days (neasest day)
Bill Balls (Bil	



- It is given that $\sin A = \frac{\sqrt{5}}{3}$ and $\sin B = \frac{1}{\sqrt{5}}$, where the angles A and B are both acute.
 - (a) (i) Show that the exact value of $\cos B = \frac{2}{\sqrt{5}}$.

[1 mark]

(ii) Hence show that the exact value of $\sin 2B$ is $\frac{4}{5}$.

[2 marks]

(b) (i) Show that the exact value of $\sin(A-B)$ can be written as $p(5-\sqrt{5})$, where p is a rational number.

[4 marks]

(ii) Find the exact value of $\cos(A-B)$ in the form $r+s\sqrt{5}$, where r and s are rational numbers.

[3 marks]

QUESTION PART	Answer space for question 5	
REFERENCE		1
5)	$\int S = \chi = \int (SS)^2 - I^2$	OR SIN'S + (0)'B = 1
ai)	h (B) x = 2	(1)2+(0182=1
	$\chi(z)$	$\frac{ Ol \sin^2 8^2 + (\cos^2 8^2 = 1)}{\left(\frac{1}{\sqrt{5}}\right)^2 + (\cos^2 8^2 = 1)}$
	$\frac{\text{CosB} = \frac{2}{2} \text{ (as reg)}}{\sqrt{5}}$	$\frac{1}{5} + \cos^2 B^2 = 1$ $\cos^2 B = 4/5$
	72	CO52B=4/5
_ii)	sin 28 = Zsin B cos B	61 B = 4 = 2
	= 2/1/2	V5 J5
	$= \frac{2}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} \right) \left(\frac{2}{\sqrt{2}} \right)$	
	= 4 (as reg)	
	5	

Tours tou	
QUESTION PART REFERENCE	Answer space for question 5
bi)	$COJA = \sqrt{1 - \left(\frac{55}{3}\right)^2} = \sqrt{1 - \frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{13}$
	$\sin A = \sqrt{3} \qquad \sin B = \frac{1}{\sqrt{3}} \qquad \cos B = \frac{2}{\sqrt{3}}$
	sin (A-B) = sin A COIB - COIASIN B
	$ \frac{3}{3} \left(\frac{2}{55} \right) - \left(\frac{2}{3} \right) \left(\frac{1}{55} \right) $
_	= 255 - 2 = 255-2 x 355
	355 355 355
	= 6(5)-655
	9(5)
	= 30-655 = 6(5-55)
	45 45
	$\frac{15}{5}\left(5-55\right)$
•	p=7/15 /3
ii)	COS(A-B) = COSA COSB + SÍN ASINB
	$= \left(\frac{2}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{5}{3}\right)\left(\frac{1}{\sqrt{5}}\right)$
	= 4 + 55 = 4+55 x 355 355 355 355 355
	$= 12\sqrt{5} + 3(5) = 12\sqrt{5} + 15$ $9(5) \qquad 45$
	9(5) 45
	$\frac{=455+1}{15} OR \frac{1}{3} + 455$
	r='/3, s=4/15
	Turn over N



The line l_1 passes through the point A(0, 6, 9) and the point B(4, -6, -11).

The line
$$l_2$$
 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$.

(a) The acute angle between the lines l_1 and l_2 is θ .

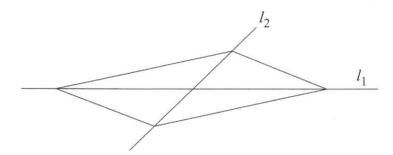
Find the value of $\cos\theta$ as a fraction in its lowest terms.

[5 marks]

(b) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection.

[5 marks]

(c) The points C and D lie on line l_2 such that ACBD is a parallelogram.



The length of AB is three times the length of CD.

Find the coordinates of the points C and Q.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 6
(a)	$a = \begin{pmatrix} 4 - 0 \\ -6 - 6 \\ -11 - 9 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix}$ $b = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$
	a.b = 4 x 3 + -12 x - 5 + - 20 x l
	= 12 + 60 - 20 = (52)
	$ a = \sqrt{(4)^2 + (-12)^2 + (-20)^2} = 4\sqrt{35}$
	$ b = \sqrt{(3)^2 + (-5)^2 + (1)^2} = \sqrt{35}$
	1a1/61 = 4 J35 x J35 = (140)
	$\cos \theta = a.6 = 52 = 13$
	1 a 11 b 140 35



REFERENCE = LZ	
b) $\begin{pmatrix} 0 \\ 6 \\ q \end{pmatrix}$ + $\begin{pmatrix} 4 \\ -n \\ -20 \end{pmatrix}$ = $\begin{pmatrix} -1 \\ s \\ -2 \end{pmatrix}$ + $\begin{pmatrix} 3 \\ -s \\ 1 \end{pmatrix}$	
0 + 4 m = -1 + 3 x 0	
6-12M = 5-5% (2)	
$9 20\mu = -2 + 2$ 3	
using O and O - 4 ps - 3 L = -1 O(x3)	
-12m +5L = -1 @	
+ 12m - 9x = -3	
-4X = -4	
L = 1	
4M-3(1)=-1	
4 _M = 2	
M' = 1/2	
check in 3 -> 9-20(1/2)=-2+1	
9-10 = -2+1	
: intersect	
when $\lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$	
$\left(\frac{-2}{2}\right) = \left(\frac{2}{2}, 0, -1\right)$	
check, when $\mu = 1/2 \rightarrow \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$	
Turn ove	





OUESTION PART REFERENCE Answer space for question 6
c) length AB (1,) = $\int 4^2 + 12^2 + 20^2 = 4\sqrt{35}$ or $\sqrt{560}$ length CD (12) = $\int \sqrt{560}$ or $\int \sqrt{35}$
length CD (12) = 1 5560 OR 4 535
3 3
midpoint of CD is (2,0,-1) (M)
length CM = 2 535
3
$C = \begin{pmatrix} -1 + 3\lambda \\ 5 + -5\lambda \end{pmatrix} \qquad M = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$
$\begin{pmatrix} S + - S \lambda \\ -2 + \lambda \end{pmatrix}$
$CM = \sqrt{(-1+3\lambda-2)^2 + (5-5\lambda)^2 + (-2+\lambda+1)^2}$
$\frac{2}{3}\sqrt{35} = \sqrt{(-3+3/)^2 + (5-5/)^2 + (-1+1)^2}$
7
$\frac{2}{3}\sqrt{35} = 9 - 18\lambda + 9\lambda^{2} + 25 + 50\lambda + 25\lambda^{2} + 1 - 2\lambda + \lambda^{2}$
``
$\frac{2}{3}\sqrt{35} = \sqrt{35} + \sqrt{2} - 70 + \sqrt{35} $ (2)
7
352 ² - 702 + 35 = 140
31522 - 6302+315=140
31522-6302+175=0 (+35)
9 2 - 18 2 + 5 = 0
$(3\lambda - 1)(3\lambda - 5) = 0$
L= 1/3
when L=1/3:-
C = (0, 10/3, -5/3) $D = (4, -10/3, -1/3)$



QUESTION PART REFERENCE	Answer space for question 6
REFERENCE	
-	
-	
	`,





7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2 - 6t}}{4}, \quad y = \frac{e^{3t}}{3t}, \quad t \neq 0$$

(a) Find the exact value of $\frac{dy}{dx}$ at the point on C where $t = \frac{2}{3}$.

[5 marks]

(b) Show that
$$x = \frac{4 - e^{2 - 6t}}{4}$$
 can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{(1 - x)}}$.

(c) Hence find the Cartesian equation of C, giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

QUESTION PART REFERENCE	Answer space for question 7
70)	$x = 4 - e^{z-6t}$, $y = e^{3t}$
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
	$\frac{dx}{dt} = \frac{3}{3}e^{2-6t}$ $\frac{dy}{dt} = \frac{3}{3}e^{2-6t}$
	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
	$= 3 t e^{3t} - e^{3t} \times 2$
	$\frac{dy}{dx} = \frac{2(3te^{3t} - e^{3t})}{9t^2e^{2-6t}}$
	when $t = \frac{1}{2}$, $\frac{dy}{dx} = \frac{2(3(^{1}/_{3})e^{3(^{1}/_{3})} - e^{3(^{1}/_{3})}}{9(^{1}/_{3})e^{2(^{1}/_{3})}}$
	$= 2(2e^{2}-e^{2}) = 1-e^{4}$



QUESTION PART REFERENCE	Answer space for question 7
6)	DC = 4 - e ^{2-6t}
3)	4
	$4x = 4 - e^{z-6t}$
	$e^{2-6t} = 4 - 4x$
	$e^{2} \times e^{-6t} = 4 - 4 \times$
	$e^2 = 4 - 4x$
	e 6+
-	$e^{6t} = e^2 \qquad (5)$
-	4 - 4x
-	$\sqrt{e^{6t}} = e^2$
-	$e^{3t} = e = e (as reg)$
-	
	5451-x 25(1-x)
()	y=e3+
	$y = e^{3t}$ $e^{3t} = e$ $2\sqrt{(1-x)}$ $3t \ln e = \ln \left(e\right)$ $2\sqrt{(1-x)}$
	3+ = 1/2 - 1/2 J(1-x)
	$y = \frac{e}{2\sqrt{(1-x)}}$ $3t = 1 - \ln 2\sqrt{(1-x)}$
_	1-1/25(1-x)
	= e
	(2x5(1-x) (+-1/25(1-x))
	when $f(x) = 2J(1-x)$
	Turn over ▶



- 8 It is given that $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$.
 - (a) By writing $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$ as $2\tan\theta = 3x$, use implicit differentiation to show that $\frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{k}{4+9x^2}$, where k is an integer.

[3 marks]

(b) Hence solve the differential equation

$$9y(4+9x^2)\frac{\mathrm{d}y}{\mathrm{d}x} = \csc 3y$$

given that x=0 when $y=\frac{\pi}{3}$. Give your answer in the form g(y)=h(x) .

[7 marks]

QUESTION PART REFERENCE	Answer space for question 8	
8a)	$\theta = \tan^{-1}\left(\frac{3x}{2}\right)$	
		Fan 0 = 32
	2 han 0 = 3x	
	Zsec 20 d0 = 3	1 2 x 4 3 x
	dα	30
	d0 = 3	2
	$\frac{d\theta}{dx} = 3$	$Sec \theta = \frac{1}{2}$
		$Sec \theta = \frac{1}{\omega \theta} = \frac{1}{2}$ $\sqrt{4+9x^2}$
	$d\theta = 3$	$\int \sec \theta = \sqrt{4+9\chi^2}$
	$\frac{dx}{dx} = 2\left(\frac{4+9x^2}{4}\right)$	2
		Sec 2 0 = 4 + 9x2
	= 12 2(4+9x²)	4
	d0 = 6	
	dx 4+9x2	
	K=6	



ib) $9y(4+9x^2) dy = GSE(3y)$ $\int 9y dy = \int 1 dx$ $\int 9y \sin 3y dy = 1 \int 6 dx$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $\int 9y \sin 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $\int 9y \cos 3y - \int -3 \cos 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $-3y \cos 3y + \int 3 \cos 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $-3y \cos 3y + \int 3 \cos 3y dy = \frac{1}{6} \int 4x^2 + 2x^2$ $-3y \cos 3y + \int 3 \cos 3y = \frac{1}{6} \int 4x^2 + 2x^2$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \int 4x^2 + 2x^2$	QUESTION PART REFERENCE	Answer space for question 8
$\int \frac{9y \sin 3y}{6} dy = \frac{1}{6} \int \frac{6}{4+9x^{2}} dx$ $\int \frac{9y \sin 3y}{6} dy = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{4+9x^{2}}{6} \int \frac{4\pi^{2}}{3x} dy = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dy = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dy = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3\cos 3y}{3y} dx = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3\cos 3y}{3y} dx = \frac{1}{6} \int \frac{4\pi^{2}}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3\cos 3y}{3y} dx + \int \frac{3x}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3\cos 3y}{3y} dx + \int \frac{3x}{3x} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3y \cos 3y}{3y} dx + \int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3x}{3} \int \frac{3x}{3y} dx + C$ $\int \frac{3x}{3} \int \frac{3x}{3} dx + C$ $\int \frac{3x}{3} \int \frac{3x}{$	6)	$\frac{9y(4+9x^2)}{dx} = 6 \sec 3y$
$\int 9y \sin 3y dy = \frac{1}{6} \tan^{-1}(3x) + C$ $U = 9y \qquad dv = \sin 3y$ $dy = 9 \qquad dy$ $dy \qquad v = -\frac{1}{3} \cos 3y $ $- 3y \cos 3y - \int -3\cos 3y dy = \frac{1}{6} \tan^{-1}(3x) + C$ $- 3y \cos 3y + \int 3\cos 3y dy = \frac{1}{6} \tan^{-1}(3x) + C$ $- 3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1}(3x) + C$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1}(3x) + C$ $2c = 0, 4Aex \ y = \frac{\pi}{3}$ $5in \left(3(\pi/3)\right) - 3(\pi/3)\cos (3(\pi/3)) = \frac{1}{6} \tan^{-1}(0) + C$		$\int \frac{9y}{\cos 2y} dy = \int \frac{1}{4+9x^2} dx$
$u = 9y \qquad dv = \sin 3y$ $du = 9 \qquad dy$ $-3y \cos 3y - \left(-3\cos 3y dy\right) = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $-3y \cos 3y + \int 3\cos 3y dy = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $x = 0, \text{When } y = \frac{\pi}{3}$ $\sin \left(\frac{3}{3} \left(\frac{\pi}{3}\right)\right) - \frac{3}{3} \left(\frac{\pi}{3}\right) \cos \left(\frac{3}{3} \left(\frac{\pi}{3}\right)\right) = \frac{1}{6} \tan^{-1} \left(0\right) + C$		$\int \frac{9ij \sin 3y}{6} \frac{dy}{6} = 1 \int \frac{6}{4 + 9x^2} \frac{dx}{6}$
$-3y \cos 3y - \left(-3\cos 3y dy = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $-3y \cos 3y + \int 3\cos 3y dy = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + C$ $2c = 0, \text{When } y = \frac{\pi}{3}$ $\sin \left(\frac{3(\pi/3)}{3(\pi/3)}\right) - \frac{3(\pi/3)}{3(\pi/3)} \cos \left(\frac{3(\pi/3)}{3(\pi/3)}\right) = \frac{1}{6} \tan^{-1} (0) + C$		$\int \frac{9y \sin 3y}{6} dy = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$
$-3y \cos 3y + \int 3\cos 3y dy = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) + C$ $-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) + C$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) + C$ $2c = 0, \text{When } y = \frac{\pi}{3}$ $5in \left(3 \left(\frac{\pi}{3} \right) \right) - 3 \left(\frac{\pi}{3} \right) \cos \left(3 \left(\frac{\pi}{3} \right) \right) = \frac{1}{6} \tan^{-1} (0) + C$		$\begin{array}{cccc} U = 9y & dv = \sin 3y \\ dy & = 9 & dy \\ dy & v = -\frac{1}{3}(0) 3y \end{array}$
$-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1}(\frac{3x}{2}) + C$ $\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1}(\frac{3x}{2}) + C$ $\cot 2x + C$ $\cot 3(\frac{\pi}{3}) - 3(\frac{\pi}{3}) \cos (3(\frac{\pi}{3})) = \frac{1}{6} \tan^{-1}(0) + C$		- 3y cos 3y - (-3 cos 3y dy = 1 tan " (3x) + c
		- Jy cos sy + S 3 cos sy dry = 1/6 tan (3x) + C
$5c=0$, when $y=\frac{\pi}{3}$ $5in(3(\frac{\pi}{3}))-3(\frac{\pi}{3})\cos(3(\frac{\pi}{3}))=\frac{1}{6}\tan^{-1}(0)+C$		- 3y cos 3y + sin 3y = 1/6 tan' (3x) + C
Sin (3(7/3)) -3(7/3) cos (3(7/s)) = 1/2 har 1(0)+C		sin 3y - 3y 601 3y = 1 tan' (3x) + c
JIN 11 - TI COST = C		$5c=0$, when $y=\frac{\pi}{3}$ $5in(3(\frac{\pi}{3}))-3(\frac{\pi}{3})\cos(3(\frac{\pi}{3}))=\frac{1}{6}har^{-1}(0)+C$ $sin \pi - \pi cos \pi = c$
$0 + \pi = C$ $50, \sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1} \left(\frac{3x}{2}\right) + \pi$		$0 + \pi = C$





QUESTION PART REFERENCE	Answer space for question 8
52.5	
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END OF QUESTIONS

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