

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

ANSWERS

Candidate signature

A-level MATHEMATICS

Unit Pure Core 4

Friday 17 June 2016

Afternoon

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\frac{19x - 3}{(1 + 2x)(3 - 4x)}$ in the form $\frac{A}{1 + 2x} + \frac{B}{3 - 4x}$.

[3 marks]

(b) (i) Find the binomial expansion of $\frac{19x - 3}{(1 + 2x)(3 - 4x)}$ up to and including the term in x^2 .

[7 marks]

(ii) State the range of values of x for which this expansion is valid.

[1 mark]

QUESTION
PART
REFERENCE

Answer space for question 1

$$1a) \quad 19x - 3 = A(3 - 4x) + B(1 + 2x)$$

$$x = \frac{3}{4} \rightarrow 19\left(\frac{3}{4}\right) - 3 = B\left(1 + 2\left(\frac{3}{4}\right)\right)$$

$$5\frac{3}{4} - 3 = 1\frac{1}{2}B$$

$$4\frac{5}{4} = 1\frac{1}{2}B$$

$$B = \frac{9}{2} \text{ OR } 4.5$$

$$x = -\frac{1}{2} \rightarrow 19\left(-\frac{1}{2}\right) - 3 = A\left(3 - 4\left(-\frac{1}{2}\right)\right)$$

$$-\frac{19}{2} - 3 = 5A$$

$$-\frac{25}{2} = 5A$$

$$A = -\frac{5}{2} \text{ OR } -2.5$$

$$\frac{19x - 3}{(1 + 2x)(3 - 4x)} = \frac{-2.5}{(1 + 2x)} + \frac{4.5}{(3 - 4x)}$$

$$= \frac{-5}{2(1 + 2x)} + \frac{9}{2(3 - 4x)}$$



QUESTION
PART
REFERENCE

Answer space for question 1

$$b) \quad -\frac{5}{2} (1+2x)^{-1} + \frac{9}{2} (3-4x)^{-1}$$

$$-\frac{5}{2} (1+2x)^{-1} = -\frac{5}{2} \left[1 + (-1)(2x) + \frac{(-1)(-2)(2x)^2}{2} \right]$$

$$= -\frac{5}{2} (1 - 2x + 4x^2)$$

$$\frac{9}{2} (3-4x)^{-1} = \frac{9}{2} (3)^{-1} \left(1 - \frac{4}{3}x\right)^{-1}$$

$$= \frac{3}{2} \left[1 + (-1)\left(-\frac{4}{3}x\right) + \frac{(-1)(-2)\left(-\frac{4}{3}\right)^2}{2} \right]$$

$$= \frac{3}{2} \left(1 + \frac{4}{3}x + \frac{16}{9}x^2 \right)$$

$$-\frac{5}{2} (1 - 2x + 4x^2) + \frac{3}{2} \left(1 + \frac{4}{3}x + \frac{16}{9}x^2 \right)$$

$$-\frac{5}{2} + 5x - 10x^2 + \frac{3}{2} + 2x + \frac{8}{3}x^2$$

$$-1 + 7x - \frac{22}{3}x^2$$

$$ii) \quad \text{valid for } |2x| < 1$$

$$|x| < \frac{1}{2}$$

$$\text{OR } |-\frac{4}{3}x| < 1$$

$$|x| < \frac{3}{4}$$

$$\therefore \underline{-\frac{1}{2} < x < \frac{1}{2}}$$

Turn over ►



2

By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3 \cos 2\theta - 5 \cos \theta + 2 = 0$$

in the interval $0^\circ < \theta < 360^\circ$, giving your answers to the nearest 0.1° .

[5 marks]

QUESTION
PART
REFERENCE

Answer space for question 2

$$2) \quad 3 \cos 2\theta - 5 \cos \theta + 2 = 0$$

$$0 < \theta < 360^\circ$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$3(2 \cos^2 \theta - 1) - 5 \cos \theta + 2 = 0$$

$$6 \cos^2 \theta - 3 - 5 \cos \theta + 2 = 0$$

$$6 \cos^2 \theta - 5 \cos \theta - 1 = 0$$

$$(6 \cos \theta + 1)(\cos \theta - 1) = 0$$

$$6 \cos \theta + 1 = 0 \quad \text{OR} \quad \cos \theta - 1 = 0$$

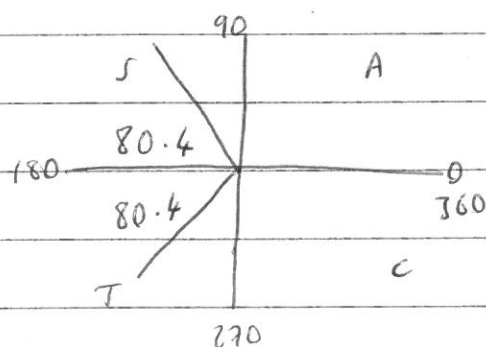
$$\cos \theta = -1/6$$

$$\cos \theta = 1$$

$$\theta = 99.59406\dots, \quad "$$

$$\theta = 0^\circ \rightarrow \text{out of range}$$

$$260.4059\dots$$



$$\theta = \underline{99.6^\circ}, \underline{260.4^\circ} \text{ (nearest } 0.1^\circ)$$



QUESTION
PART
REFERENCE

Answer space for question 2

Turn over ►



3 (a) Express $\frac{3 + 13x - 6x^2}{2x - 3}$ in the form $Ax + B + \frac{C}{2x - 3}$.

[4 marks]

(b) Show that $\int_3^6 \frac{3 + 13x - 6x^2}{2x - 3} dx = p + q \ln 3$, where p and q are rational numbers.

[4 marks]

QUESTION
PART
REFERENCE

Answer space for question 3

$$\begin{array}{r}
 3a) \quad \frac{3 + 13x - 6x^2}{2x - 3} \rightarrow \quad \begin{array}{r} -3x + 2 \\ 2x - 3 \overline{) -6x^2 + 13x + 3} \\ \underline{-6x^2 + 9x} \\ 0 + 4x + 3 \\ \underline{4x - 6} \\ 0 + 9 \end{array}
 \end{array}$$

$$\frac{3 + 13x - 6x^2}{2x - 3} = -3x + 2 + \frac{9}{2x - 3}$$

$$A = -3, B = 2, C = 9$$

$$b) \int_3^6 -3x + 2 + \frac{9}{2x - 3} dx$$

$$\left[-\frac{3x^2}{2} + 2x + \frac{9}{2} \ln(2x - 3) \right]_3^6$$

$$\left(-\frac{3(6)^2}{2} + 2(6) + \frac{9}{2} \ln(2(6) - 3) \right) - \left(-\frac{3(3)^2}{2} + 2(3) + \frac{9}{2} \ln(2(3) - 3) \right)$$

$$\left(-42 + \frac{9}{2} \ln 9 \right) - \left(-\frac{15}{2} + \frac{9}{2} \ln 3 \right)$$

$$-\frac{69}{2} + \frac{9}{2} \ln 3^2 - \frac{9}{2} \ln 3$$

$$-\frac{69}{2} + \frac{18}{2} \ln 3 - \frac{9}{2} \ln 3 = -\frac{69}{2} + \frac{9}{2} \ln 3$$

$$p = -\frac{69}{2}, q = \frac{9}{2}$$



Answer space for question 3

[illegible]

- 4 The mass of radioactive atoms in a substance can be modelled by the equation

$$m = m_0 k^t$$

where m_0 grams is the initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

- (a) (i) A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days.

Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

[1 mark]

- (ii) A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.

Calculate the initial mass of this sample, giving your answer to the nearest gram.

[2 marks]

- (b) The half-life of a radioactive substance is the time it takes for a mass of m_0 to reduce to a mass of $\frac{1}{2}m_0$.

A sample of radioactive vanadium reduced in mass from exactly 10 grams to 8.106 grams in 100 days.

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

[4 marks]

QUESTION
PART
REFERENCE

Answer space for question 4

4a(i)

$$M = M_0 k^t$$

$$M_0 = 24, M = 12, t = 8$$

$$12 = 24 \times k^8$$

$$\frac{1}{2} = k^8$$

$$k = \sqrt[8]{1/2} = 0.9170040... = 0.917004 \text{ (6dp)}$$

ii)

$$M = M_0 \times 0.917004^t \quad M = 1, t = 60$$

$$1 = M_0 \times 0.917004^{60}$$

$$M_0 = \frac{1}{0.917004^{60}} = 181.0198477$$

$$= 181 \text{ g (nearest g)}$$



QUESTION
PART
REFERENCE

Answer space for question 4

b)

$$M = M_0 k^t$$

$$M_0 = 10, \quad M = 8.106, \quad t = 100$$

$$8.106 = 10 \times k^{100}$$

$$k = \sqrt[100]{\frac{8.106}{10}} = 0.9979023$$

$$M = 5, \quad M_0 = 10, \quad k = 0.9979023$$

$$5 = 10 \times 0.9979023^t$$

$$0.5 = 0.9979023^t$$

$$\log 0.5 = t \log 0.9979023$$

$$t = \frac{\log 0.5}{\log 0.9979023}$$

$$t = 330.0852928$$

$$t = \underline{\underline{330 \text{ days}}} \text{ (nearest day)}$$



- 5 It is given that $\sin A = \frac{\sqrt{5}}{3}$ and $\sin B = \frac{1}{\sqrt{5}}$, where the angles A and B are both acute.

(a) (i) Show that the exact value of $\cos B = \frac{2}{\sqrt{5}}$.

[1 mark]

(ii) Hence show that the exact value of $\sin 2B$ is $\frac{4}{5}$.

[2 marks]

- (b) (i) Show that the exact value of $\sin(A - B)$ can be written as $p(5 - \sqrt{5})$, where p is a rational number.

[4 marks]

- (ii) Find the exact value of $\cos(A - B)$ in the form $r + s\sqrt{5}$, where r and s are rational numbers.

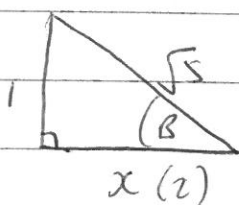
[3 marks]

QUESTION
PART
REFERENCE

Answer space for question 5

5)

ai)



$$x = \sqrt{(\sqrt{5})^2 - 1^2}$$

$$x = 2$$

$$\text{or } \sin^2 B + \cos^2 B = 1$$

$$\left(\frac{1}{\sqrt{5}}\right)^2 + \cos^2 B = 1$$

$$\cos B = \frac{2}{\sqrt{5}} \quad (\text{as req})$$

$$\frac{1}{5} + \cos^2 B = 1$$

$$\cos^2 B = \frac{4}{5}$$

ii)

$$\sin 2B = 2 \sin B \cos B$$

$$= 2 \left(\frac{1}{\sqrt{5}}\right) \left(\frac{2}{\sqrt{5}}\right)$$

$$= \frac{4}{5} \quad (\text{as req})$$

$$\cos B = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$



QUESTION
PART
REFERENCE

Answer space for question 5

$$b) \cos A = \sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2} = \sqrt{1 - \frac{5}{9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$\sin A = \frac{\sqrt{5}}{3}, \quad \sin B = \frac{1}{\sqrt{5}}, \quad \cos A = \frac{2}{3}, \quad \cos B = \frac{2}{\sqrt{5}}$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{\sqrt{5}}\right) - \left(\frac{2}{3}\right)\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{2\sqrt{5}}{3\sqrt{5}} - \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5} - 2}{3\sqrt{5}} \times \frac{3\sqrt{5}}{3\sqrt{5}}$$

$$= \frac{6(5) - 6\sqrt{5}}{9(5)}$$

$$= \frac{30 - 6\sqrt{5}}{45} = \frac{6(5 - \sqrt{5})}{45}$$

$$= \frac{2}{15}(5 - \sqrt{5})$$

$$p = \frac{2}{15}$$

$$ii) \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \left(\frac{2}{3}\right)\left(\frac{2}{\sqrt{5}}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{\sqrt{5}}\right)$$

$$= \frac{4}{3\sqrt{5}} + \frac{\sqrt{5}}{3\sqrt{5}} = \frac{4 + \sqrt{5}}{3\sqrt{5}} \times \frac{3\sqrt{5}}{3\sqrt{5}}$$

$$= \frac{12\sqrt{5} + 3(5)}{9(5)} = \frac{12\sqrt{5} + 15}{45}$$

$$= \frac{4\sqrt{5}}{15} + \frac{1}{3} \quad \text{OR} \quad \frac{1}{3} + \frac{4\sqrt{5}}{15}$$

$$r = \frac{1}{3}, \quad s = \frac{4}{15}$$



- 6 The line l_1 passes through the point $A(0, 6, 9)$ and the point $B(4, -6, -11)$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$.

- (a) The acute angle between the lines l_1 and l_2 is θ .

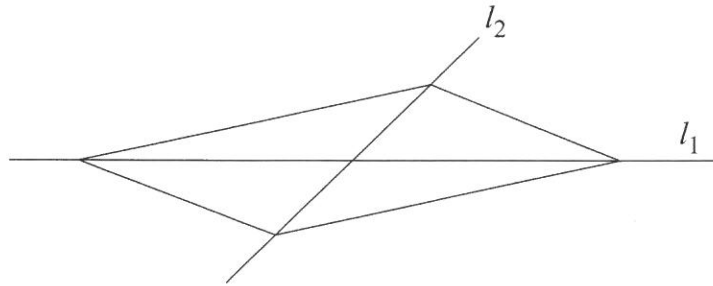
Find the value of $\cos \theta$ as a fraction in its lowest terms.

[5 marks]

- (b) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection.

[5 marks]

- (c) The points C and D lie on line l_2 such that $ACBD$ is a parallelogram.



The length of AB is three times the length of CD .

Find the coordinates of the points C and D .

[5 marks]

QUESTION
PART
REFERENCE

Answer space for question 6

$$6a) \quad \mathbf{a} = \begin{pmatrix} 4-0 \\ -6-6 \\ -11-9 \end{pmatrix} = \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = 4 \times 3 + (-12) \times (-5) + (-20) \times 1 \\ = 12 + 60 - 20 = 52$$

$$|\mathbf{a}| = \sqrt{(4)^2 + (-12)^2 + (-20)^2} = 4\sqrt{35}$$

$$|\mathbf{b}| = \sqrt{(3)^2 + (-5)^2 + (1)^2} = \sqrt{35}$$

$$|\mathbf{a}| |\mathbf{b}| = 4\sqrt{35} \times \sqrt{35} = 140$$

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{52}{140} = \frac{13}{35}$$



QUESTION
PART
REFERENCE

Answer space for question 6

$$b) \quad l_1 = l_2$$

$$\begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$$

$$0 + 4\mu = -1 + 3\lambda \quad (1)$$

$$6 - 12\mu = 5 - 5\lambda \quad (2)$$

$$9 - 20\mu = -2 + \lambda \quad (3)$$

$$\text{using (1) and (2)} \rightarrow 4\mu - 3\lambda = -1 \quad (1 \times 3)$$

$$\begin{array}{r} -12\mu + 5\lambda = -1 \quad (2) \\ + \\ 12\mu - 9\lambda = -3 \\ \hline -4\lambda = -4 \end{array}$$

$$\lambda = 1$$

$$4\mu - 3(1) = -1$$

$$4\mu = 2$$

$$\mu = \frac{1}{2}$$

$$\text{check in (3)} \rightarrow 9 - 20\left(\frac{1}{2}\right) = -2 + 1$$

$$9 - 10 = -2 + 1 \quad \checkmark$$

\therefore intersect

$$\text{when } \lambda = 1 \rightarrow \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} = \underline{\underline{(2, 0, -1)}}$$

$$\text{check, when } \mu = \frac{1}{2} \rightarrow \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 4 \\ -12 \\ -20 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

Turn over ►



QUESTION
PART
REFERENCE

Answer space for question 6

$$c) \text{ length AB (L)} = \sqrt{4^2 + 12^2 + 20^2} = 4\sqrt{35} \text{ OR } \sqrt{560}$$

$$\text{length CD (L)} = \frac{1}{3} \sqrt{560} \text{ OR } \frac{4}{3} \sqrt{35}$$

midpoint of CD is $(2, 0, -1)$ (M)

$$\text{length CM} = \frac{2}{3} \sqrt{35}$$

$$C = \begin{pmatrix} -1 + 3\lambda \\ 5 - 5\lambda \\ -2 + \lambda \end{pmatrix} \quad M = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

$$CM = \sqrt{(-1 + 3\lambda - 2)^2 + (5 - 5\lambda)^2 + (-2 + \lambda + 1)^2}$$

$$\frac{2}{3} \sqrt{35} = \sqrt{(-3 + 3\lambda)^2 + (5 - 5\lambda)^2 + (-1 + \lambda)^2}$$

$$\frac{2}{3} \sqrt{35} = \sqrt{9 - 18\lambda + 9\lambda^2 + 25 + 50\lambda + 25\lambda^2 + 1 - 2\lambda + \lambda^2}$$

$$\frac{2}{3} \sqrt{35} = \sqrt{35\lambda^2 - 70\lambda + 35} \quad (*)$$

$$35\lambda^2 - 70\lambda + 35 = \frac{140}{9}$$

$$315\lambda^2 - 630\lambda + 315 = 140$$

$$315\lambda^2 - 630\lambda + 175 = 0 \quad (\div 35)$$

$$9\lambda^2 - 18\lambda + 5 = 0$$

$$(3\lambda - 1)(3\lambda - 5) = 0$$

$$\lambda = \frac{1}{3}, \quad \lambda = \frac{5}{3}$$

when $\lambda = \frac{1}{3} :-$

$$C = \left(0, \frac{10}{3}, -\frac{5}{3}\right)$$

when $\lambda = \frac{5}{3} :-$

$$D = \left(4, -\frac{10}{3}, -\frac{1}{3}\right)$$



QUESTION
PART
REFERENCE

Answer space for question 6

Turn over ►



- 7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2-6t}}{4}, \quad y = \frac{e^{3t}}{3t}, \quad t \neq 0$$

- (a) Find the exact value of $\frac{dy}{dx}$ at the point on C where $t = \frac{2}{3}$.

[5 marks]

- (b) Show that $x = \frac{4 - e^{2-6t}}{4}$ can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{1-x}}$.

[2 marks]

- (c) Hence find the Cartesian equation of C , giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

QUESTION
PART
REFERENCE

Answer space for question 7

7a) $x = \frac{4 - e^{2-6t}}{4}, \quad y = \frac{e^{3t}}{3t}$

$x = 1 - \frac{1}{4}e^{2-6t}$ $y = \frac{1}{3}t^{-1}e^{3t} \rightarrow u = \frac{1}{3}t^{-1}, v = e^{3t}$

$\frac{dx}{dt} = \frac{3}{2}e^{2-6t}$ $\frac{dy}{dt} = \frac{-1}{3}t^{-2}e^{3t} + \frac{3e^{3t}}{3t} = \frac{3te^{3t} - e^{3t}}{3t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \frac{3te^{3t} - e^{3t}}{3t^2} \times \frac{2}{3e^{2-6t}}$

$\frac{dy}{dx} = \frac{2(3te^{3t} - e^{3t})}{9t^2e^{2-6t}}$

when $t = \frac{2}{3}$, $\frac{dy}{dx} = \frac{2(3(\frac{2}{3})e^{3(\frac{2}{3})} - e^{3(\frac{2}{3})})}{9(\frac{2}{3})^2e^{2-6(\frac{2}{3})}}$

$= \frac{2(2e^2 - e^2)}{4e^{-2}} = \frac{1}{2}e^4$



QUESTION
PART
REFERENCE

Answer space for question 7

$$b) \quad x = \frac{4 - e^{2-6t}}{4}$$

$$4x = 4 - e^{2-6t}$$

$$e^{2-6t} = 4 - 4x$$

$$e^2 \times e^{-6t} = 4 - 4x$$

$$\frac{e^2}{e^{6t}} = 4 - 4x$$

$$e^{6t} = \frac{e^2}{4 - 4x} \quad (\sqrt{\quad})$$

$$\sqrt{e^{6t}} = \sqrt{\frac{e^2}{4 - 4x}}$$

$$e^{3t} = \frac{e}{\sqrt{4} \sqrt{1-x}} = \frac{e}{2\sqrt{1-x}} \quad (\text{as req})$$

$$c) \quad y = \frac{e^{3t}}{3t}$$

$$e^{3t} = \frac{e}{2\sqrt{1-x}}$$

$$3t \ln e = \ln \left(\frac{e}{2\sqrt{1-x}} \right)$$

$$3t = \ln e - \ln 2\sqrt{1-x}$$

$$y = \frac{e}{2\sqrt{1-x}} \cdot \frac{1}{1 - \ln 2\sqrt{1-x}}$$

$$3t = 1 - \ln 2\sqrt{1-x}$$

$$= \frac{e}{(2\sqrt{1-x})(1 - \ln 2\sqrt{1-x})}$$

$$\text{When } f(x) = 2\sqrt{1-x}$$

Turn over ►



8 It is given that $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$.

- (a) By writing $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$ as $2 \tan \theta = 3x$, use implicit differentiation to show that $\frac{d\theta}{dx} = \frac{k}{4 + 9x^2}$, where k is an integer.

[3 marks]

- (b) Hence solve the differential equation

$$9y(4 + 9x^2) \frac{dy}{dx} = \operatorname{cosec} 3y$$

given that $x = 0$ when $y = \frac{\pi}{3}$. Give your answer in the form $g(y) = h(x)$.

[7 marks]

QUESTION
PART
REFERENCE

Answer space for question 8

8a) $\theta = \tan^{-1}\left(\frac{3x}{2}\right)$

$$2 \tan \theta = 3x$$

$$2 \sec^2 \theta \frac{d\theta}{dx} = 3$$

$$\frac{d\theta}{dx} = \frac{3}{2 \sec^2 \theta}$$

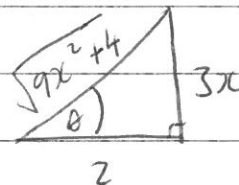
$$\frac{d\theta}{dx} = \frac{3}{2 \left(\frac{4 + 9x^2}{4} \right)}$$

$$= \frac{12}{2(4 + 9x^2)}$$

$$\frac{d\theta}{dx} = \frac{6}{4 + 9x^2}$$

$$k = 6$$

$$\tan \theta = \frac{3x}{2}$$



$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{2}{\sqrt{4 + 9x^2}}}$$

$$\sec \theta = \frac{\sqrt{4 + 9x^2}}{2}$$

$$\sec^2 \theta = \frac{4 + 9x^2}{4}$$



QUESTION
PART
REFERENCE

Answer space for question 8

$$b) \quad 9y(4+9x^2) \frac{dy}{dx} = \operatorname{cosec} 3y$$

$$\int \frac{9y}{\operatorname{cosec} 3y} dy = \int \frac{1}{4+9x^2} dx$$

$$\int 9y \sin 3y dy = \frac{1}{6} \int \frac{6}{4+9x^2} dx$$

$$\int 9y \sin 3y dy = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$$

| | |
|---------------------|----------------------------|
| $u = 9y$ | $\frac{dv}{dy} = \sin 3y$ |
| $\frac{du}{dy} = 9$ | $v = -\frac{1}{3} \cos 3y$ |

$$-3y \cos 3y - \int -3 \cos 3y dy = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$-3y \cos 3y + \int 3 \cos 3y dy = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$-3y \cos 3y + \sin 3y = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$\sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + C$$

$$x=0, \text{ when } y = \pi/3$$

$$\sin\left(3\left(\frac{\pi}{3}\right)\right) - 3\left(\frac{\pi}{3}\right) \cos\left(3\left(\frac{\pi}{3}\right)\right) = \frac{1}{6} \tan^{-1}(0) + C$$

$$\sin \pi - \pi \cos \pi = C$$

$$0 + \pi = C$$

$$\text{So, } \sin 3y - 3y \cos 3y = \frac{1}{6} \tan^{-1}\left(\frac{3x}{2}\right) + \pi$$

Turn over ►



