AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	Written Solutions
Forename(s)	
Candidate signature	

AS MATHEMATICS

Unit Pure Core 2

Wednesday 25 May 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Find $\int \left(\frac{36}{x^2} + ax\right) dx$, where a is a constant.

[3 marks]

(b) Hence, given that $\int_1^3 \left(\frac{36}{x^2} + ax\right) dx = 16$, find the value of the constant a.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

1a) $\int (36x^{-2} + ax)\, dx$

$\frac{36x^{-1}}{-1} + \frac{ax^2}{2} + c$

$-36x^{-1} + \frac{ax^2}{2} + c$

b) $\left[-36x^{-1} + \frac{ax^2}{2}\right]_1^3 = 16$

$\left(-36(3)^{-1} + \frac{a(3)^2}{2}\right) - \left(-36(1)^{-1} + \frac{a(1)^2}{2}\right) = 16$

$\left(-12 + \frac{9a}{2}\right) - \left(-36 + \frac{a}{2}\right) = 16$

$-12 + 36 + \frac{9a}{2} - \frac{a}{2} = 16$

$24 + 4a = 16$

$4a = -8$

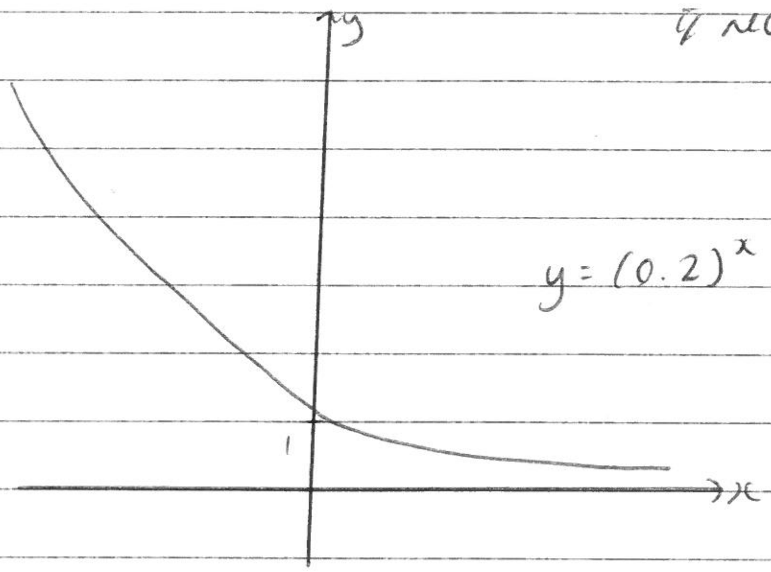
$a = -2$

Do not write outside the box

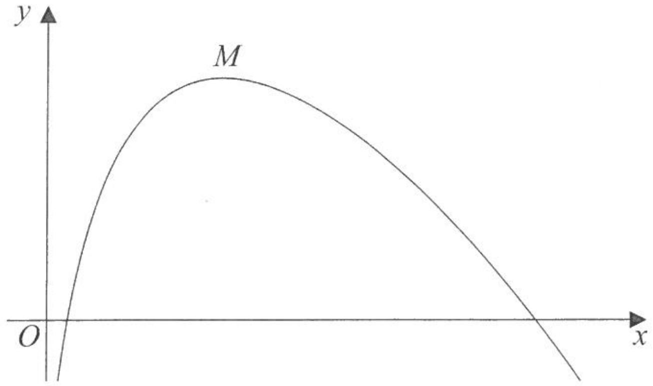
2 (a) Sketch the graph of $y = (0.2)^x$, indicating the value of the intercept on the y-axis.
[2 marks]

(b) Use logarithms to solve the equation $(0.2)^x = 4$, giving your answer to three significant figures.
[2 marks]

(c) Describe the geometrical transformation that maps the graph of $y = (0.2)^x$ onto the graph of $y = 5^x$.
[1 mark]

QUESTION PART REFERENCE	**Answer space for question 2**
	* use table of values if necessary
2a)	
b)	$(0.2)^x = 4$
	$x \log 0.2 = \log 4$
	$x = \frac{\log 4}{\log 0.2}$
	$x = -0.8613531161$
	$x = -0.861$ (3sf)
c)	$y = (0.2)^x \rightarrow y = \left(\frac{1}{5}\right)^x - y = 5^{-x}$
	$y = 5^{-x} \rightarrow y = 5^x$ is a reflected in y axis
	(also use graph to help)

3 The diagram shows a curve with a maximum point M.



The curve is defined for $x > 0$ by the equation

$$y = 6x^{\frac{1}{2}} - x - 3$$

(a) Find $\dfrac{dy}{dx}$.

[2 marks]

(b) **Hence** find the y-coordinate of the maximum point M.

[3 marks]

(c) Find an equation of the normal to the curve at the point $P(4, 5)$.

[3 marks]

(d) It is given that the normal to the curve at P, when translated by the vector $\begin{bmatrix} k \\ 0 \end{bmatrix}$, passes through the point M. Find the value of the constant k.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 3**
3a)	$y = 6x^{\frac{1}{2}} - x - 3$ $\dfrac{dy}{dx} = 3x^{-1/2} - 1$
b)	maximum point $\rightarrow \dfrac{dy}{dx} = 0$, $3x^{-1/2} - 1 = 0$ $3x^{-1/2} = 1$ $\dfrac{3}{\sqrt{x}} = 1$, $x = 9$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 3

$y = 6(9)^{\frac{1}{2}} - (9) - 3$

$= 18 - 9 - 3$

$\underline{\underline{y = 6}}$

c) gradient of tangent at $(4, 5)$:-

$\frac{dy}{dx} = 3(4)^{-1/2} - 1$

$= \frac{3}{2} - 1$

$\frac{dy}{dx} = \frac{1}{2}$ → gradient of normal is -2

$y - 5 = -2(x - 4)$

$y - 5 = -2x + 8$

$\underline{y = -2x + 13}$

d) translated $\begin{pmatrix} k \\ 0 \end{pmatrix}$ so $x = 9 - k$ $y = 6$

$6 = -2(9 - k) + 13$

$6 = -18 + 2k + 13$

$6 = -5 + 2k$

$2k = 11$

$\underline{\underline{k = 5.5}}$

Do not write outside the box

4 An arithmetic series has first term a and common difference d.

The sum of the first 21 terms is 168.

(a) Show that $a + 10d = 8$.

[3 marks]

(b) The sum of the second term and the third term is 50.

The nth term of the series is u_n.

(i) Find the value of u_{12}.

[4 marks]

(ii) Find the value of $\sum_{n=4}^{21} u_n$.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 4
4a)	$S_{21} = 168$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$
	$168 = \frac{21}{2}(2a + 20d)$
	$168 = 21(a + 10d)$
	$168 = 21a + 210d$ $\quad (\div 21)$
	$a + 10d = 8$ (as req)
bi)	$S_2 + S_3 = 50$
	$a + d + a + 2d = 50$
	$2a + 3d = 50$
	$a + 10d = 8$ $\quad (\times 2)$
	$-\ 2a + 20d = 16$
	$2a + 3d = 50$ $\quad 2a + 3(-2) = 50$
	$17d = -34$ $\quad 2a - 6 = 50$
	$d = -2$ $\quad 2a = 56,\ a = 28$

1 0

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 4

$U_{12} = a + 11d$

$= 28 + 11 \times -2$

$= 28 - 22$

$U_{12} = \underline{6}$

ii) $\sum_{n=4}^{21} U_n \rightarrow S_{21} - S_3$

$S_3 = 28 + 50 = 78$

$S_{21} = 168$

$\sum_{n=4}^{21} U_n = 168 - 78$

$= \underline{90}$

Do not write outside the box

5 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_2^{11} \sqrt{x^2+9}\,dx$. Give your answer to one decimal place.

[4 marks]

(b) Describe the geometrical transformation that maps the graph of $y = \sqrt{x^2+9}$ onto the graph of :

(i) $y = 5 + \sqrt{x^2+9}$;

[2 marks]

(ii) $y = 3\sqrt{x^2+1}$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

5a) $\int_2^{11} \sqrt{x^2+9}\, dx$ $\quad h = \frac{11-2}{3} = 3$

x	2	5	8	11
y	$\sqrt{13}$	$\sqrt{34}$	$\sqrt{73}$	$\sqrt{130}$

$= \frac{3}{2}\left(\sqrt{13} + \sqrt{130} + 2(\sqrt{34} + \sqrt{73})\right)$

$= 65.63582521$

$= \underline{65.6}$ (1 dp)

bi) $y = 5 + \sqrt{x^2+9} \rightarrow f(x) + 5$

translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

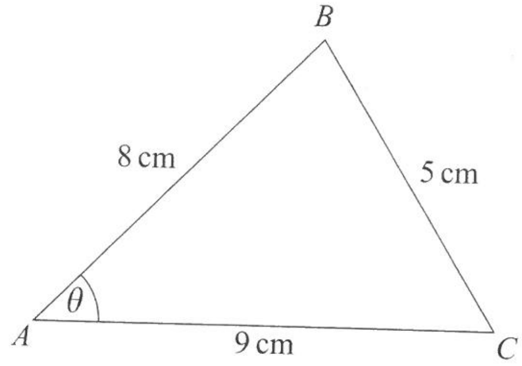
ii) $y = 3\sqrt{x^2+1} \rightarrow \sqrt{9} \times \sqrt{x^2+1} = \sqrt{9(x^2+1)}$

$= \sqrt{9x^2+9}$

$= \sqrt{(3x)^2+9}$

$f(3x) \rightarrow$ stretch, scale factor $1/3$ in x direction

6 The diagram shows a triangle ABC.



The lengths of AB, BC and AC are $8\,\text{cm}$, $5\,\text{cm}$ and $9\,\text{cm}$ respectively.

Angle BAC is θ **radians**.

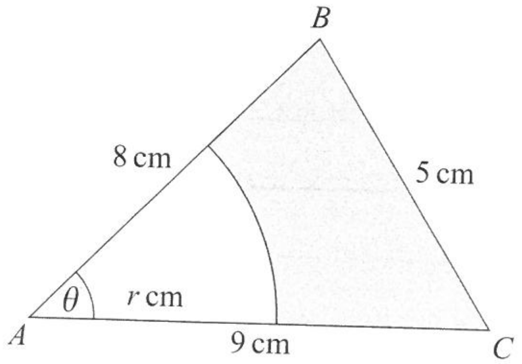
(a) Show that $\theta = 0.586$, correct to three significant figures.

[3 marks]

(b) Find the area of triangle ABC, giving your answer, in cm^2, to three significant figures.

[2 marks]

(c) A circular sector, centre A and radius r cm, is removed from triangle ABC. The remaining shape is shown shaded in the diagram below.



Given that the area of the sector removed is equal to the area of the shaded shape, find the perimeter of the shaded shape. Give your answer in cm to three significant figures.

[6 marks]

QUESTION PART REFERENCE	Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

6a) $\cos A = \dfrac{8^2 + 9^2 - 5^2}{2 \times 8 \times 9}$

$\cos A = \dfrac{120}{144}$

$A = \cos^{-1}\left(\dfrac{120}{144}\right)$

$A = 33.55730976$ * radians *

$\theta = 0.5856855435$

$\underline{\underline{\theta = 0.586}}$ (3sf)

b) Area $= \frac{1}{2} ab\sin C$

$= \frac{1}{2} \times 8 \times 9 \times \sin 0.586$

$= 19.89974874 = \underline{\underline{19.9cm^2}}$ (3sf)

c) Area of sector is half area of triangle

Area of sector $= \frac{1}{2}$ of 19.9

$= 9.949874371$

$\frac{1}{2} r^2 \theta = 9.949874371$

$\frac{1}{2} \times r^2 \times 0.586 = 9.94987..$

$r^2 = \dfrac{9.94987..}{\frac{1}{2} \times 0.586}$

$r^2 = 33.9586158$, $r = \underline{5.827402131}$

Arc length $= r\theta = 5.827402.. \times 0.586$

$= 3.414857669$

Turn over ▶

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 6

Perimeter $= (8-r) + 9(-r) + \cancel{5} + \text{arc length}$

$= 2.171216904 + 3.171216904 + 5$ ~~...~~

$+ 3.4148...$

$= 13.757233$

$= \underline{13.8}$ cm (3sf)

Do not write outside the box

7 (a) The expression $(1-2x)^5$ can be written in the form

$$1 + px + qx^2 + rx^3 + 80x^4 - 32x^5$$

By using the binomial expansion, or otherwise, find the values of the coefficients p, q and r.

[3 marks]

(b) Find the value of the coefficient of x^{10} in the expansion of $(1-2x)^5(2+x)^7$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

7a) $(1-2x)^5 = ({}^5C_0)(1)^5(-2x)^0 = 1$

$({}^5C_1)(1)^4(-2x)^1 = 5(1)(-2x) = -10x$

$({}^5C_2)(1)^3(-2x)^2 = 10(1)(4x^2) = 40x^2$

$({}^5C_3)(1)^2(-2x)^3 = 10(1)(-8x^3) = -80x^3$

$({}^5C_4)(1)^1(-2x)^4 = 5(1)(16x^4) = 80x^4$

$({}^5C_5)(1)^0(-2x)^5 = 1(1)(-32x^5) = -32x^5$

$1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

$p = -10$, $q = 40$, $r = -80$

b) $(2+x)^7 = ({}^7C_0)(2)^7(x)^0 = 128$

$({}^7C_1)(2)^6(x)^1 = 448x$

$({}^7C_2)(2)^5(x)^2 = 672x^2$

$({}^7C_3)(2)^4(x)^3 = 560x^3$

$({}^7C_4)(2)^3(x)^4 = 280x^4$

$({}^7C_5)(2)^2(x)^5 = 84x^5$

$({}^7C_6)(2)^1(x)^6 = 14x^6$

$({}^7C_7)(2)^0(x)^7 = x^7$

$128 + 448x + 672x^2 + 560x^3 + 280x^4 + 84x^5 + 14x^6 + x^7$

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 7

$(1-2x)^5(2+x)^7$

$(1-10x+40x^2-80x^3+80x^4-32x^5)(128+448x+$
$672x^2+560x^3+280x^4+84x^5$
$+14x^6+x^7)$

$x^{10} \rightarrow -80x^3 \times x^7 = -80x^{10}$
$80x^4 \times 14x^6 = 1120x^{10}$
$-32x^5 \times 84x^5 = -2688x^{10}$

$-80x^{10} + 1120x^{10} - 2688x^{10} = -1648x^{10}$

$\underline{-1648}$

8 (a) (i) Given that $4\sin x + 5\cos x = 0$, find the value of $\tan x$.

[2 marks]

(ii) Hence solve the equation $(1 - \tan x)(4\sin x + 5\cos x) = 0$ in the interval $0° \leqslant x \leqslant 360°$, giving your values of x to the nearest degree.

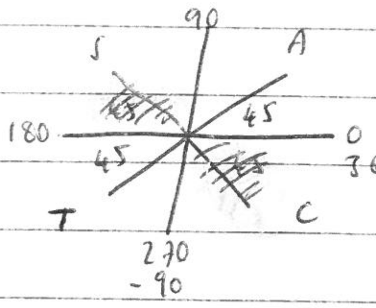
[3 marks]

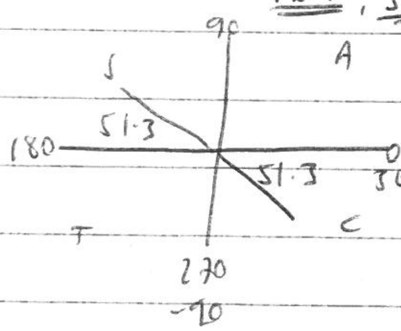
(b) By first showing that $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta}$ can be expressed in the form $p + q\cos\theta$, where p and q are integers, find the least possible value of $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta}$.

State the exact value of θ, in radians in the interval $0 \leqslant \theta < 2\pi$, at which this least value occurs.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 8
8ai)	$4\sin x + 5\cos x = 0$
	$4\sin x = -5\cos x$
	$\dfrac{\sin x}{\cos x} = -\dfrac{5}{4}$
	$\tan x = -\dfrac{5}{4}$
	$\boxed{\text{Range } 0 \leq x \leq 360°}$
ii)	$(1 - \tan x)(4\sin x + 5\cos x) = 0$
	$1 - \tan x = 0$ OR $\tan x = -\dfrac{5}{4}$
	$\tan x = 1$
	$x = +45, 225°$
	not in range
	$x = -51.340191$
	$129°, 309°$





Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

$x = \underline{129°}, \underline{45°}, \underline{225°}, \underline{309°}$

b) $\dfrac{16 + 9\sin^2\theta}{5 - 3\cos\theta}$

$\dfrac{16 + 9(1 - \cos^2\theta)}{5 - 3\cos\theta} = \dfrac{16 + 9 - 9\cos^2\theta}{5 - 3\cos\theta}$

$\dfrac{(25 - 9\cos^2\theta)}{(5 - 3\cos\theta)} \times \dfrac{(5 + 3\cos\theta)}{(5 + 3\cos\theta)}$

$\dfrac{(25 - 9\cos^2\theta)(5 + 3\cos\theta)}{(5 - 3\cos\theta)(5 + 3\cos\theta)}$

$\dfrac{(\cancel{25 - 9\cos^2\theta})(5 + 3\cos\theta)}{\cancel{25 - 9\cos^2\theta}} = \underline{5 + 3\cos\theta}$

$p = 5, \; q = 3$

least value when $\cos\theta = -1 \rightarrow$

$5 + 3\cos\theta = 5 + 3(-1)$

$= \underline{\underline{2}}$

occurs at $\underline{\underline{\theta = \pi}}$ between $0 \leq \theta < 2\pi$

Turn over ▶

Do not write outside the box

9 (a) Given that $\log_3 c = m$ and $\log_{27} d = n$, express $\frac{\sqrt{c}}{d^2}$ in the form 3^y, where y is an expression in terms of m and n.

[4 marks]

(b) Show that the equation

$$\log_4(2x+3) + \log_4(2x+15) = 1 + \log_4(14x+5)$$

has only one solution and state its value.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 9

9a) $\log_3 c = m \qquad \log_{27} d = n$

$3^m = c \qquad 27^n = d$

$\sqrt{c} = c^{\frac{1}{2}} \qquad (3^3)^n = d$

$c^{\frac{1}{2}} = (3^m)^{\frac{1}{2}} \qquad 3^{3n} = d$

$\boxed{c^{\frac{1}{2}} = 3^{\frac{m}{2}}} \qquad d^2 = (3^{3n})^2$

$\boxed{d^2 = 3^{6n}}$

$\frac{\sqrt{c}}{d^2} = \frac{3^{m/2}}{3^{6n}} = 3^{m/2} \div 3^{6n}$

$= 3^{m/2 - 6n}$

b) $\log_4(2x+3) + \log_4(2x+15) = 1 + \log_4(14x+5)$

$\log_4(2x+3)(2x+15) = \log_4 4 + \log_4(14x+5)$

$\log_4(4x^2+36x+45) = \log_4 4(14x+5)$

$\log_4(4x^2+36x+45) = \log_4(56x+20)$

$4x^2+36x+45 = 56x+20$

$4x^2 - 20x + 25 = 0$

$(2x-5)(2x-5) = 0$

$2x - 5 = 0$ (twice

$x = 5/2$

2 2