AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ANSWERS

Forename(s)

Candidate signature

AS MATHEMATICS

Unit Pure Core 1 Non-Calculator

A

Wednesday 18 May 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The line AB has equation $5x + 3y + 3 = 0$.

(a) The line AB is parallel to the line with equation $y = mx + 7$.

Find the value of m.

[2 marks]

(b) The line AB intersects the line with equation $3x - 2y + 17 = 0$ at the point B.

Find the coordinates of B.

[3 marks]

(c) The point with coordinates $(2k + 3, 4 - 3k)$ lies on the line AB.

Find the value of k.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 1**
1a)	$5x + 3y + 3 = 0$
	$3y = -5x - 3$
	$y = -\frac{5}{3}x - 1$
	$m = -5/3$
b)	$5x + 3y = -3 \quad (\times 2)$
	$3x - 2y = -17 \quad (\times 3)$
	$10x + 6y = -6$
+	$9x - 6y = -51$
	$19x = -57$
	$x = -3 \quad \rightarrow$ sub in
	$5(-3) + 3y = -3$
	$(-3, 4) = B \qquad -15 + 3y = -3 \qquad y = 4$
	$3y = 12$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1
c)	$(2k+3,\ 4-3k)$ → sub in
	$5(2k+3)+3(4-3k)+3=0$
	$10k+15+12-9k+3=0$
	$k+30=0$
	$\underline{k=-30}$

Turn over ▶

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2 (a) Simplify $(3\sqrt{5})^2$.

[1 mark]

(b) Express $\dfrac{(3\sqrt{5})^2+\sqrt{5}}{7+3\sqrt{5}}$ in the form $m+n\sqrt{5}$, where m and n are integers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

2a) $(3\sqrt{5})^2 = 3\times\sqrt{5}\times 3\times\sqrt{5}$

$= 9\sqrt{25}$

$= 9\times 5 = \underline{45}$

b) $\dfrac{(3\sqrt{5})^2+\sqrt{5}}{7+3\sqrt{5}}$

$\dfrac{45+\sqrt{5}}{7+3\sqrt{5}} \times \dfrac{7-3\sqrt{5}}{7-3\sqrt{5}}$

$\dfrac{315-135\sqrt{5}+7\sqrt{5}-3\sqrt{25}}{49-21\sqrt{5}+21\sqrt{5}-45}$

$\dfrac{315-128\sqrt{5}-15}{4}$

$\dfrac{300-128\sqrt{5}}{4} = \underline{\underline{75-32\sqrt{5}}}$

Do not write outside the box

3 (a) (i) Express $x^2 - 7x + 2$ in the form $(x-p)^2 + q$, where p and q are rational numbers.
[2 marks]

(ii) Hence write down the minimum value of $x^2 - 7x + 2$.
[1 mark]

(b) Describe the geometrical transformation which maps the graph of $y = x^2 - 7x + 2$ onto the graph of $y = (x-4)^2$.
[3 marks]

QUESTION PART REFERENCE	Answer space for question 3
3ai)	$x^2 - 7x + 2 = (x - \frac{7}{2})^2 - \frac{49}{4} + 2$
	$= (x - \frac{7}{2})^2 - \frac{41}{4}$
ii)	$-\frac{41}{4}$
b)	from $(x - \frac{7}{2})^2 - \frac{41}{4}$ to $(x-4)^2$
	$(x - 3\frac{1}{2})^2 - \frac{41}{4}$ to $(x-4)^2$
	Translation $\begin{pmatrix} \frac{1}{2} \\ +41/4 \end{pmatrix}$

4 The polynomial $p(x)$ is given by $p(x) = x^3 - 5x^2 - 8x + 48$.

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $p(x)$.

[2 marks]

(ii) Express $p(x)$ as a product of three linear factors.

[3 marks]

(b) (i) Use the Remainder Theorem to find the remainder when $p(x)$ is divided by $x - 2$.

[2 marks]

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + bx + c) + r$, where b, c and r are integers.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 4
4ai)	$p(x) = x^3 - 5x^2 - 8x + 48$
	$p(-3) = (-3)^3 - 5(-3)^2 - 8(-3) + 48$
	$= -27 - 45 + 24 + 48$
	$= 0$
	$\therefore (x+3)$ is a factor of $p(x)$
ii)	$(x+3)(x^2 + bx + c) = x^3 - 5x^2 - 8x + 48$
	$(x+3)(x^2 - 8x + 16)$
	* compare coefficients of x^2 or x
	or division

$$
\begin{array}{r}
x^2 - 8x + 16 \\
x+3 \overline{)\, x^3 - 5x^2 - 8x + 48} \\
-\ \underline{x^3 + 3x^2 \qquad\qquad} \\
-8x^2 - 8x \qquad \\
-\ \underline{-8x^2 - 24x \qquad} \\
0 + 16x + 48 \\
-\ \underline{+16x + 48} \\
0
\end{array}
$$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

bi) $p(2) = (2)^3 - 5(2)^2 - 8(2) + 48$

$= 8 - 20 - 16 + 48$

$= -12 - 16 + 48$

$= -28 + 48$

$= 20$

Remainder is $\underline{\underline{20}}$

ii)

$$\begin{array}{r} x^2 - 3x - 14 \\ x-2 \overline{) x^3 - 5x^2 - 8x + 48} \\ -\; x^3 - 2x^2 \quad\quad\quad\quad \\ \hline 0 - 3x^2 - 8x \quad\quad \\ -\; -3x^2 + 6x \quad\quad \\ \hline 0 - 14x + 48 \\ -\; -14x + 28 \\ \hline 0 + 20 \end{array}$$

$\underline{(x-2)(x^2 - 3x - 14) + 20}$

5 A circle with centre $C(5, -3)$ passes through the point $A(-2, 1)$.

(a) Find the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = k$$

[3 marks]

(b) Given that AB is a diameter of the circle, find the coordinates of the point B.

[2 marks]

(c) Find an equation of the tangent to the circle at the point A, giving your answer in the form $px + qy + r = 0$, where p, q and r are integers.

[5 marks]

(d) The point T lies on the tangent to the circle at A such that $AT = 4$.

Find the length of CT.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 5

5a) $C(5, -3)$ $A(-2, 1)$

$(x-5)^2 + (y+3)^2 = k$

distance AC (radius) $= \sqrt{7^2 + 4^2}$

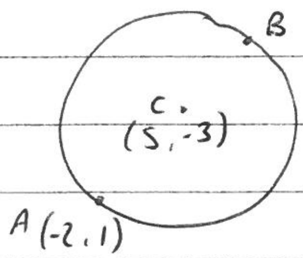
$= \sqrt{49+16}$

radius $= \sqrt{65}$

so $k = (\sqrt{65})^2 = 65$

$\underline{(x-5)^2 + (y+3)^2 = 65}$

b)



(x) $-2 \xrightarrow{+7} 5 \xrightarrow{+7} 12$

(y) $1 \xrightarrow{-4} -3 \xrightarrow{-4} -7$

$\underline{B\ (12, -7)}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

c) gradient of tangent is perpendicular to normal

gradient of AC (normal) $= \frac{1 - -3}{-2 - 5} = \frac{4}{-7}$

gradient of tangent is $\frac{7}{4}$

through point $A(-2, 1)$

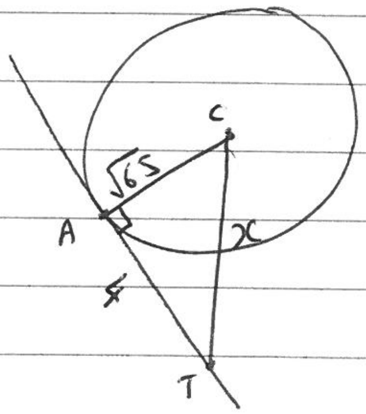
$y - 1 = \frac{7}{4}(x + 2)$

$4y - 4 = 7(x + 2)$

$4y - 4 = 7x + 14$

$\underline{7x - 4y + 18 = 0}$

d)



$x^2 = (\sqrt{65})^2 + 4^2$

$= 65 + 16$

$x^2 = 81$

$\underline{x = 9}$

so $CT = \underline{\underline{9}}$

6 (a) A curve has equation $y = 8 - 4x - 2x^2$.

(i) Find the values of x where the curve crosses the x-axis, giving your answer in the form $m \pm \sqrt{n}$, where m and n are integers.

[2 marks]

(ii) Sketch the curve, giving the value of the y-intercept.

[2 marks]

(b) A line has equation $y = k(x + 4)$, where k is a constant.

(i) Show that the x-coordinates of any points of intersection of the line with the curve $y = 8 - 4x - 2x^2$ satisfy the equation

$$2x^2 + (k+4)x + 4(k-2) = 0$$

[1 mark]

(ii) Find the values of k for which the line is a tangent to the curve $y = 8 - 4x - 2x^2$.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 6

6a) crosses x axis when $y = 0$

$8 - 4x - 2x^2 = 0$

$2x^2 + 4x - 8 = 0 \quad (\div 2)$

$x^2 + 2x - 4 = 0$ formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = 2,\ c = -4$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 16}}{2}$$

$$= \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm \sqrt{4}\sqrt{5}}{2}$$

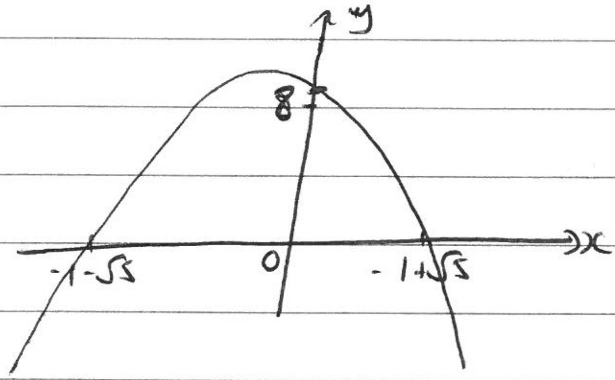
$$= \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

ii) $-ve\ x^2$ so  shape

crosses at $(0, 8)$ and $(-1+\sqrt{5}, 0)$ and $(-1-\sqrt{5}, 0)$



b i) $y = k(x+4)$ and $y = 8-4x-2x^2$

$k(x+4) = 8-4x-2x^2$

$kx + 4k = 8 - 4x - 2x^2$

$2x^2 + 4x + kx + 4k - 8 = 0$

$2x^2 + (k+4)x + 4(k-2) = 0$

ii) tangent so only <u>one</u> solution

$\therefore\ b^2 - 4ac = 0$ $\quad a = 2,\ b = k+4,$ $\quad c = 4(k-2)$

$(k+4)^2 - 4(2)(4k-8) = 0$

$k^2 + 8k + 16 - 8(4k-8) = 0$

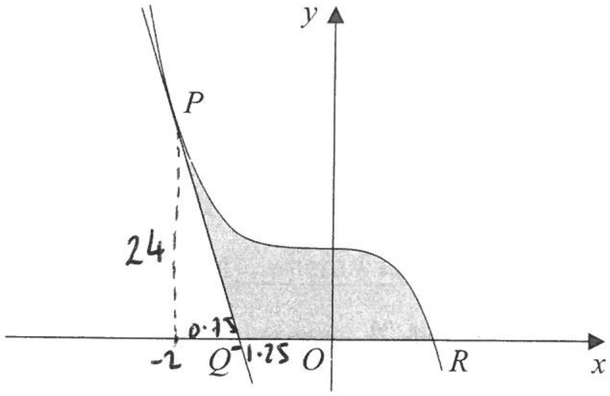
$k^2 + 8k + 16 - 32k + 64 = 0$

$k^2 - 24k + 80 = 0$

$(k-20)(k-4) = 0$ $\quad$ <u>$k=20$</u> or <u>$k=4$</u>

Turn over ▶

7 The diagram shows the sketch of a curve and the tangent to the curve at P.



The curve has equation $y = 4 - x^2 - 3x^3$ and the point $P(-2, 24)$ lies on the curve. The tangent at P crosses the x-axis at Q.

(a) (i) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$.

[5 marks]

(ii) Hence find the x-coordinate of Q.

[1 mark]

(b) (i) Find $\int_{-2}^{1} (4 - x^2 - 3x^3)\,dx$.

[5 marks]

(ii) The point $R(1, 0)$ lies on the curve. Calculate the area of the shaded region bounded by the curve and the lines PQ and QR.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 7
7ai)	gradient of tangent → use $\frac{dy}{dx}$ at $(-2, 24)$
	$y = 4 - x^2 - 3x^3$
	$\frac{dy}{dx} = -2x - 9x^2$
	when $x = -2$, $\frac{dy}{dx} = -2(-2) - 9(-2)^2$
	$= 4 - 36 = -32$
	gradient is -32, $P(-2, 24)$

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QUESTION PART REFERENCE	Answer space for question 7

$$y - 24 = -32(x+2)$$
$$y - 24 = -32x - 64$$
$$\underline{y = -32x - 40}$$

ii) at Q, $y = 0$

$$-32x - 40 = 0$$
$$32x = -40$$
$$x = \underline{-40/32} = -1.25$$

bi) $\int_{-2}^{1} (4 - x^2 - 3x^3)\,dx = \left[4x - \frac{x^3}{3} - \frac{3x^4}{4}\right]_{-2}^{1}$

$$= \left(4(1) - \frac{(1)^3}{3} - \frac{3(1)^4}{4}\right) - \left(4(-2) - \frac{(-2)^3}{3} - \frac{3(-2)^4}{4}\right)$$
$$= \left(4 - \frac{1}{3} - \frac{3}{4}\right) - \left(-8 + \frac{8}{3} - \frac{48}{4}\right)$$
$$= \left(4 - \frac{1}{3} - \frac{3}{4}\right) - \left(-8 + \frac{8}{3} - 12\right)$$
$$= 4 - \frac{1}{3} - \frac{8}{3} - \frac{3}{4} - (-20)$$
$$= 4 - \frac{9}{3} - \frac{3}{4} + 20$$
$$= 24 - 3 - 3/4 = \underline{20\,1/4}$$

ii) shaded area = area under curve − area of $\triangle$

area of $\triangle$ $= \frac{24 \times 0.75}{2} = 9$

shaded area $= 20\,1/4 - 9$

$= \underline{11\,1/4}$

Turn over ▶

1 5

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8 The gradient, $\frac{dy}{dx}$, at the point (x, y) on a curve is given by

$$\frac{dy}{dx} = 54 + 27x - 6x^2$$

(a) (i) Find $\frac{d^2y}{dx^2}$.

[2 marks]

(ii) The curve passes through the point $P\left(-1\frac{1}{2}, 4\right)$.

Verify that the curve has a minimum point at P.

[4 marks]

(b) (i) Show that at the points on the curve where y is decreasing

$$2x^2 - 9x - 18 > 0$$

[2 marks]

(ii) Solve the inequality $2x^2 - 9x - 18 > 0$.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 8
8ai)	$\frac{dy}{dx} = 54 + 27x - 6x^2$
	$\frac{d^2y}{dx^2} = \underline{27 - 12x}$
ii)	when $x = -1\frac{1}{2}$,
	$\frac{dy}{dx} = 54 + 27(-3/2) - 6(-3/2)^2$
	$= 54 - 40.5 - 6(9/4)$
	$= 54 - 40.5 - \frac{54}{4}$
	$= 54 - 40.5 - 13.5$
	$\frac{dy}{dx} = 0$ $\therefore$ stationary point at P

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QUESTION PART REFERENCE

Answer space for question 8

when $x = -1\frac{1}{2}$,

$$\frac{d^2y}{dx^2} = 27 - 12(-1\tfrac{1}{2})$$
$$= 27 + 18$$
$$= 45$$

$\frac{d^2y}{dx^2} > 0$ $\therefore$ minimum point at P.

b i) y is decreasing when $\frac{dy}{dx} < 0$

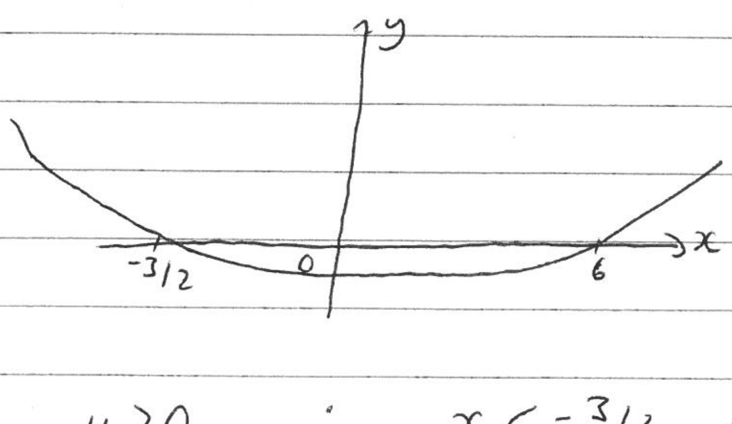
$$54 + 27x - 6x^2 < 0$$
$$6x^2 - 27x - 54 > 0 \quad (\div 3)$$
$$2x^2 - 9x - 18 > 0 \quad \text{(as required)}$$

ii) $2x^2 - 9x - 18 = 0$

$(2x + 3)(x - 6) = 0$

$2x + 3 = 0$ or $x - 6 = 0$

$x = -3/2$ $\qquad$ $x = 6$



$y > 0$, $\therefore$ $x < -3/2$ or $x > 6$